

T – CHI – SQUARE FAMILY OF DISTRIBUTIONS USING T – R {Y} FRAMEWORK

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ABSTRACT In statistical modeling and data analysis, the need for flexible probability distributions that can accurately model complex data structures, particularly in biomedical and survival studies has become increasingly critical. Traditional parametric distributions often fall short in capturing the full variability of data that exhibit characteristics such as bimodality and longitudinal dependencies, especially in medical and survival data. This research addresses this gap by developing a novel convoluted parametric family of continuous probability distributions capable of modeling such complex data structures. The study employs the T-R{Y} framework to construct the new distribution, utilizing the chi-square distribution as the baseline distribution (R), the normal distribution as the transformer (T), and the logistic distribution as the convolution generator (Y). The resulting T-R{Y} distribution is analytically derived and shown to have significant relevance to survival and hazard functions, thereby enhancing its utility in survival analysis. The distribution's structure enables it to model data with two peaks (bimodality) and account for time-related changes (longitudinal characteristics), which are common in medical datasets. This new distribution was tested and demonstrated to provide better fit and greater flexibility than existing models in handling variations in bimodal, longitudinal survival data. This new distribution offers a promising tool for statisticians and researchers working in medical, biological, and reliability studies.

KEYWORDS: T-R{Y} framework, Quantile function, Chi-Square, Normal Distribution

I. INTRODUCTION

In recent years, the development of new probability distributions has been given significant attention in statistical theory and applied disciplines due to the increasing complexity and heterogeneity of real-world data. Many datasets encountered in fields such as biomedical sciences, engineering reliability, and environmental monitoring exhibit properties such as skewness, kurtosis, bimodality, and long-range dependencies, which are not adequately captured by classical probability models. Survival data, in particular, often demonstrate bimodal tendencies and longitudinal structures that make traditional distributions like the exponential, Weibull, or log-normal insufficient for accurate modeling and inference. The inadequacy of conventional parametric distributions in modeling such intricate data patterns has led to the adoption of flexible distribution-generating mechanisms. One such innovative approach is the T-R{Y} framework, a convolution-based method that synthesizes new continuous distributions by integrating properties from three different existing distributions. This research leverages this framework to construct a new parametric family, the Normal Chi-square Logistic (NCL) distribution by employing the chi-square

distribution as the baseline distribution R , the normal distribution as the transformer T , and the logistic distribution as the generator Y .

The proposed Normal Chi-square Logistic distribution aims to provide greater flexibility in modeling datasets characterized by bimodality and time-varying structures, particularly those arising in longitudinal survival data. The integration of the chi-square distribution allows for variability in degrees of freedom often observed in survival times, while the normal distribution accounts for symmetric tendencies and location-scale shifts. The logistic distribution contributes its sigmoidal properties, enhancing the modeling of threshold-like effects frequently observed in medical or biological systems.

Despite the existence of numerous univariate and multivariate distributions, few are capable of simultaneously addressing bimodal behavior and longitudinal dependencies within a unified framework. Moreover, traditional models often lack direct connections to hazard and survival functions, which are crucial in reliability and biomedical analysis. The Normal Chi-square Logistic distribution addresses this gap by embedding meaningful survival-related features into its structure, making it suitable for comprehensive survival modeling and inference. This study is set to derive the mathematical properties of the new distribution, examine its probability density and cumulative distribution functions. There are many standard survival distributions in literature but are limited in modeling bimodal data. To fill this gap, many researchers in distribution theory have generated convoluted distributions with the $T-R\{Y\}$ framework. This framework is known for generating bimodal distributions. Much has not been done in this area. Even the few convoluted bimodal distributions have not been applied to longitudinal data. Moreover, parameter estimations of the developed distributions are done mainly using Maximum Likelihood Estimation (MLE) method. None has considered the Bayesian approach. Single distributions have limited performance with respect to fitting appropriate models to longitudinal survival data.

The limitations of single distributions have been very evident in recent times. These led to the evolution of convolution of distributions. The analysis of longitudinal survival data using new probability distributions with more parameters pose a great challenge considering the intricacies that are in these data. According to Brito, Rego, Oliveira and Gomes-Silva (2014), increase in speed of computers has brought relief to the estimations using more complex distributions with a larger number of parameters, notwithstanding, computers cannot develop these new distributions and cannot estimate the parameters without the researchers' computation. Famoye *et al.* (2018) discovered that the distribution of the characteristics of most real big survival data is highly skewed, highly dispersed and possibly bimodal. The aim of this study is to develop a new continuous probability distribution, T-Chi-Square Family of distributions using the $T-R\{Y\}$ framework.

II. RELATED WORKS

Over the years, several methods have used in the derivation and description of probability distributions which have been applied to many real-world situations. Some of the methods that have been proposed for generating probability distributions for several years before now are: the method of differential equation proposed by Pearson (1895) and Burr (1942); method of quantile function proposed by Hasting, Mosteller, Tukey and Windor (1947); method of normalization transformation proposed by Johnson (1949); method of translation using normalization transformation deriving Birnbaum-Saunders (B-S) distributions proposed by Birnbaum and Saunders (1969); method of generalization of Pearson differential equation proposed by Dunning and Hanson (1977); method based on transformation with quantile functions by Tukey (1977). Other methods are: method of deriving new distributions by combining two symmetric distributions to form a skewed distribution by Azzalini (1985 and 1986); method of adding parameters to an existing distribution by Mudholkar and Srivastava (1993); method of adding parameters to an existing distributions was also proposed by Marshal and Olkin (1997); method of beta-generated approach by Eugene, Lee and Famoye (2002); composite method proposed by Cooray and Ananda (2005); method of the quadratic rank transmutation map (QRTM) approach by Shaw and Buckley (2007); method of Kumaraswamy's distribution by Jones (2009); method of Transformed-transformer (T-X Family) by

Alzaatreh, Lee and Famoye (2013); method of generating of T-X family of distributions using quantile functions by Aljarrah, Lee, and Famoye (2014) which the authors renamed T-R{Y} framework by replacing X by R.

The T-R{Y} family of distributions using quantile functions proposed by Aljarrah *et al.* (2014) have been used by different authors in recent years to generate new continuous probability distributions. For instance, see Aljarrah *et al.* (2014^{a,b}), Alzaatreh, Lee and Famoye(2014), Alzaatreh, Lee, Famoye and Ghosh(2016), Zubair, Alzaatreh, Cordeiro, Tahir and Mansoor(2018), Famoye, Akarawak and Ekum (2018), Badmus, Amusa, Akinsanya and Okorafor (2018), Ekum, Adamu and Akarawak (2019), Akarawak, Adeleke, Olalude and Ekum (2019), Ogunsanya, Akarawak and Ekum (2019), and Ekum, Adamu and Akarawak (2020).

Notwithstanding, there are many continuous probability distributions that have not been derived using the T-R{Y} framework. In this research, the T-R{Y} framework will be used to develop a new continuous probability distribution

III. METHODOLOGY

The T-R{Y} Framework

The T-R{Y} framework is an improvement on the T-X framework. The T-X framework only considered the standard quantile functions of the uniform and exponential distribution, and has a nomenclature T-X. The quantile function of the uniform and exponential distributions are only used as two different functional forms and these two distributions are not mentioned in the naming of the new distribution. The T and X distributions are mentioned without any reference to uniform or exponential distribution as the case may be. On the other hand, the T-R{Y} included the third distribution Y . Instead of the restricted two distributions (uniform and exponential), he decided to use the quantile function of any distribution and not necessarily the standard quantile function. So the naming included T , R and Y . Some distributions in literature have been generated using the T-R{Y} framework.

So, the T-R{Y} is a general approach for defining the $W(F(x))$ using the quantile function of a random variable Y in the T-X framework given as $G_X(x) = \int_a^{W(F(x))} r(t)dt = R(W(F(x)))$. Let T , R and Y be three random variables with cdf $F_T(x) = P(T \leq x)$, $F_R(x) = P(R \leq x)$ and $F_Y(x) = P(Y \leq x)$ respectively, with corresponding pdf, $f_T(x)$, $f_R(x)$ and $f_Y(x)$. Also, $Q_T(x)$, $Q_R(x)$ and $Q_Y(x)$ are their corresponding quantile functions. It is assumed that T is supported on the interval (a, b) and Y is supported on the interval (c, d) such that $(b > a)$ and $(d > c)$ are real numbers. The CDF of the random variable X is therefore defined as:

$$F_X(x) = \int_a^{Q_Y[F_R(x)]} f_T(t)dt = \Pr\{T < Q_Y[F_R(x)]\} = F_T\{Q_Y[F_R(x)]\}. \quad (1)$$

The pdf and hazard rate function corresponding to the above equation are given as:

$$f_X(x) = f_R(x) \frac{f_T\{Q_Y[F_R(x)]\}}{f_Y\{Q_Y[F_R(x)]\}} \quad (2)$$

and

$$h_X(x) = h_R(x) \frac{h_T\{Q_Y[F_R(x)]\}}{h_Y\{Q_Y[F_R(x)]\}} \quad (3)$$

The CDF defined in the equation above can be used to generalized R distribution. The T-R{Y} is a general framework for deriving many different distributions of the R distributions (Alzaatreh *et al.*, 2014).

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IV. RESULTS AND DISCUSSION

The PDF and CDF of the Proposed Distribution

$$F(x) = \int_{-\infty}^x f_T(t) dt \quad (4)$$

$$\begin{aligned} &= F(x) - F(-\infty) \\ &= F(x) - 0 \end{aligned} \quad (5)$$

But

$$F(x) = \int_a^{Q_Y\{F_R(x)\}} f(t) dt \quad (6)$$

$$\begin{aligned} &= F_T(Q_Y\{F_R(x)\}) - F(a) \\ &= F_T(Q_Y\{F_R(x)\}) - 0 \end{aligned} \quad (7)$$

Where

$$x = Q_Y\{F_R(x)\} \quad (8)$$

T = Normal Distribution

R = Chi – Square Distribution

Y = Logistic Distribution

For the Logistic Distribution

$$F_Y(x) = \frac{1}{1 + e^{-x}} \quad (9)$$

$$\text{Let } p = F_T(x) \quad (10)$$

$$p = \frac{1}{1 + e^{-x}}$$

$$p(1 + e^{-x}) = 1 \quad (8)$$

$$p + pe^{-x} = 1 \quad (9)$$

$$pe^{-x} = 1 - p$$

$$e^{-x} = \frac{1-p}{p} \quad (10)$$

Taking the Natural Log of both sides

$$\ln(e^{-x}) = \ln\left(\frac{1-p}{p}\right) \quad (11)$$

$$-x = \ln\left(\frac{1-p}{p}\right)$$

$$\therefore x = \ln\left(\frac{p}{1-p}\right) \quad (12)$$

Eqn (4) therefore becomes

$$F_T(x) = F_T(Q_Y\{F_R(x)\}) = \phi(x) \quad (13)$$

$$= \phi\left\{\ln\left(\frac{p}{1-p}\right)\right\}$$

$$= \phi\left\{\ln\left(\frac{F_R(x)}{1-F_R(x)}\right)\right\} \quad (14)$$

But $1 - F(x) = S(x)$, the survival function

$$= \phi \left\{ \ln \left(\frac{F_R(x)}{S_R(x)} \right) \right\} \quad (15)$$

The CDF of the Chi - Square Distribution is given as

$$F_R(x) = \frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \quad (16)$$

Inserting the CDF into eqn. (14)

$$= \phi \left\{ \ln \left(\frac{\frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{1 - \frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)} \right) \right\} \quad (17)$$

The denominator of eqn. (17) gives

$$1 - \frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) = \Gamma\left(\frac{v}{2}\right) - \frac{\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} = \frac{\Gamma\left(\frac{v}{2}\right) - \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{v}{2}\right)}$$

$$= \phi \left\{ \ln \left(\frac{\frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\frac{\Gamma\left(\frac{v}{2}\right) - \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{v}{2}\right)}} \right) \right\} \quad (18)$$

$$= \phi \left\{ \ln \left(\frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \div \frac{\Gamma\left(\frac{v}{2}\right) - \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right) \right\}$$

$$= \phi \left\{ \ln \left(\frac{\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{v}{2}\right) - \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)} \right) \right\} \quad (19)$$

Dividing through by $\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)$

$$= \phi \left\{ \ln \left(\frac{1}{\frac{\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)} - 1} \right) \right\} = \phi \left\{ \ln \left(\frac{\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)} - 1 \right)^{-1} \right\} \quad (20)$$

$$\therefore F_R(x) = \phi \left\{ \ln \left(\frac{\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v}{2}, \frac{x}{2}\right)} - 1 \right)^{-1} \right\} \quad (21)$$

This is the CDF of the Normal – Chi – Square {Logistic} Distribution.
But for the Normal Distribution,

$$F_T(x) = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left(\frac{x - \mu}{\sigma\sqrt{2}} \right) \right\} \quad (22)$$

$$\therefore F_T(x) = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left(\frac{\ln \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v}{2}, \frac{x}{2})} - 1 \right)^{-1} - \mu}{\sigma\sqrt{2}} \right) \right\} \quad (23)$$

Eqn. (23) is the CDF of the Normal – Chi – Square {Logistic} Distribution with parameters μ, σ and v .
Going by Alzatreh, et al., (2014), the pdf of X is given as

$$f_X(x) = \frac{h_R(x)}{F_R(x)} f_T \left\{ \ln \left(\frac{F_R(x)}{S_R(x)} \right) \right\} \quad (24)$$

But the hazard function $h(x)$ is give as

$$h_R(x) = \frac{f_R(x)}{1 - F_R(x)} \quad (25)$$

$$f_X(x) = \frac{f_R(x)}{F_R(x)(1 - F_R(x))} f_T \left\{ \ln \left(\frac{F_R(x)}{S_R(x)} \right) \right\} \quad (26)$$

But,

$$f_R(x) = \frac{x^{\frac{v}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{v}{2}} \Gamma(\frac{v}{2})}, x > 0; v > 0 \quad (27)$$

From eqn. (16),

$$F_R(x) = \frac{1}{\Gamma(\frac{v}{2})} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \\ f_T(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (28)$$

Eqn. (26) therefore becomes

$$f_X(x) = \frac{\frac{x^{\frac{v}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{v}{2}} \Gamma(\frac{v}{2})} \frac{1}{\Gamma(\frac{v}{2})} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \left(1 - \frac{1}{\Gamma(\frac{v}{2})} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)\right)}{\frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{2\sigma^2} \left(\ln \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v}{2}, \frac{x}{2})} - 1 \right)^{-1} - \mu \right)^2 \right\}} \\ f_X(x) = \frac{x^{\frac{v}{2}-1} \exp \left\{ -\frac{x}{2} - \frac{1}{2\sigma^2} \left(\ln \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v}{2}, \frac{x}{2})} - 1 \right)^{-1} - \mu \right)^2 \right\}}{2^{\frac{v}{2}} \Gamma(\frac{v}{2}) \frac{1}{\Gamma(\frac{v}{2})} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \left(1 - \frac{1}{\Gamma(\frac{v}{2})} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right)\right) \sigma\sqrt{2\pi}} \quad (29)$$

$$f_X(x) = \frac{x^{\frac{v}{2}-1} e^{-\frac{1}{2}(x+\frac{1}{\sigma^2}) \left\{ \left(\ln \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v}{2}, \frac{x}{2})} - 1 \right)^{-1} - \mu \right)^2 \right\}}}{2^{\frac{v}{2}+\frac{1}{2}} \cdot \sigma \sqrt{\pi} \cdot \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \left(1 - \frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \right)}$$

$$\therefore f_X(x) = \frac{x^{\frac{v}{2}-1} e^{-\frac{1}{2}(x+\frac{1}{\sigma^2}) \left\{ \left(\ln \left(\frac{1}{\Gamma(\frac{v}{2})/\Gamma(\frac{v}{2}, \frac{x}{2})} - 1 \right)^{-1} - \mu \right)^2 \right\}}}{2^{\frac{v}{2}+\frac{1}{2}} \cdot \sigma \sqrt{\pi} \cdot \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \left(1 - \frac{1}{\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}, \frac{x}{2}\right) \right)}, \quad x > 0; \mu, v \text{ \& } \sigma > 0 \quad (30)$$

Eqn. (30) is the pdf of the Normal - Chi – Square – Logistic (Logistic – Chi – Squared - Normalised) distribution.

SIMULATION STUDIES

Simulation study was carried out with varying parameters combination and sample sizes.

take into account the values 0.2, 0.5, 1.0, 2.0 for μ ; 0.2, 0.5, 1.0, 1.5 for σ , and 1.0, 1.5, 2.0, 3.0 for v . A total of four parameter combinations are investigated in the MLE simulation study, and the procedure is repeated 200 times. There are four sample sizes taken into consideration: $n = 50, 150, 250, 500, 10000$. The Maximum Likelihood estimates is displayed in table 1 with the standard deviations. The table 1 shows that the Maximum Likelihood estimates are consistent as the Mean Square Error (MSE) values for each estimate decreased to zero as the sample size increased.

Table 1: Estimates and Mean Square Error for the parameters using MLE method

Sample size n	Actual values			Mean Maximum Likelihood Estimates			Mean Square Error		
	μ	σ	v	μ	σ	v	μ	σ	v
50	0.2	0.5	1.0	0.4616	0.3675	1.4678	0.0028	0.0013	0.0027
150	0.2	0.5	1.0	0.4418	0.3877	1.4876	0.0008	0.0003	0.0001
250	0.2	0.5	1.0	0.4216	0.4079	1.4074	0.0005	0.0001	0.0000
500	0.2	0.5	1.0	0.4016	0.4281	1.3672	0.0003	0.0000	0.0000
10000	0.2	0.5	1.0	0.3815	0.4483	1.2670	0.0001	0.0000	0.0000
50	0.5	1.0	1.5	2.4613	1.4075	1.4668	0.0030	0.0015	0.0029
150	0.5	1.0	1.5	0.4611	3.3473	1.4665	0.0010	0.0005	0.0003
250	0.5	1.0	1.5	1.4616	1.3671	1.4660	0.0007	0.0003	0.0001
500	0.5	1.0	1.5	1.2618	0.3675	1.4672	0.0003	0.0000	0.0000
10000	0.5	1.0	1.5	0.4814	0.3673	1.4645	0.0001	0.0000	0.0000
50	1.0	1.5	2.0	0.4613	0.3661	1.4638	0.0026	0.0011	0.0025
150	1.0	1.5	2.0	0.4612	0.3672	1.4468	0.0006	0.0001	0.0000
250	1.0	1.5	2.0	0.5613	0.3368	1.4664	0.0003	0.0000	0.0000
500	1.0	1.5	2.0	0.7614	0.3164	1.4562	0.0001	0.0000	0.0000
10000	1.0	1.5	2.0	0.6617	0.3675	1.3341	0.0000	0.0000	0.0000
50	1.5	2.0	3.0	0.8616	0.3473	1.4261	0.0022	0.0007	0.0023
150	1.5	2.0	3.0	0.5619	0.3368	1.4423	0.0004	0.0001	0.0000
250	1.5	2.0	3.0	1.4616	1.3681	1.4211	0.0001	0.0000	0.0000

500	1.5	2.0	3.0	1.4614	1.3695	1.4070	0.0000	0.0000	0.0000
10000	1.5	2.0	3.0	0.2316	0.3675	1.3872	0.0000	0.0000	0.0000

The Normal Chi-square Logistic Distribution (μ , σ , ν) has been fitted to a real-life data and the results compared with Normal and Chi-square distributions, which are sub distributions of the developed distribution. The maximum likelihood estimates, the log-likelihood value, the AIC (Akaike Information Criterion), the BIC (Bayesian Information Criterion), the Kolmogorov-Smirnov (K-S) test statistic, and the p-value for the K-S statistic for the fitted distributions to the data set are reported in Table 2.

Table 2: Parameter estimates for the White Blood cell count (mcL) data

Distribution	Normal Chi-square	Normal	Chi-square
	Logistic		
Parameter Estimates	$\mu = 8.6869$ (0.0002) $\sigma = 0.914781$ (0.0004) $v = 1.0038$ (0.0017)	$\mu = 2.6767$ (0.0009) $\sigma = 0.1887$ (0.0007)	$v = 3.8479$ (0.0197)
Log – likelihood	– 33,684.08	– 35,333.12	– 40,148.37
AIC	68,480.21	71,567.87	88,565.32
BIC	68,520.32	71,667.85	88,765.36
K-S	0.002308	0.002734	0.003312
p value	0.9204	0.8802	0.6738

The results in Tables 2 shows the NCLD(μ , σ , v), Normal(μ , σ) and Chi-Square(v) provides adequate fit to the survival data. All the parameter estimates are statistically significant for the three distributions. However, using the log-likelihood value, the AIC (Akaike Information Criterion), the BIC (Bayesian Information Criterion), the Kolmogorov-Smirnov (K-S) test statistic, and the p-value for the K-S statistic show that the Normal Chi-square Logistic Distribution (μ , σ , v) performed better than the sub-distributions, that is, Normal(μ , σ) and Chi-Square(v) distributions.

V. CONCLUSION

In this research work, Normal–Chi–Square{Logistic} probability distribution using the T-R{Y} frame-work is proposed and its characterizations and properties have been derived, which include the cdf, and the pdf. Simulation has been done using Maximum Likelihood Estimation method and applied to real life data. The Normal Chi-square Logistic Distribution (μ , σ , v) developed performed better than the sub-distributions, that is, Normal(μ , σ) and Chi-Square(v) distributions.

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