

Modified Laplace-Adomian Decomposition Method for Solving System of Fractional Partial Differential Equations

Ibrahim Ganiyu Olayiwola¹ and Taiwo Omotayo Adebayo²

¹Department of mathematics, Kwara State College of Education, Oro, Nigeria

²Department of Mathematics, University of Ilorin, Ilorin, Nigeria

Corresponding author: Ibrahim G, Olayiwola (e-mail: goibrahim63@gmail.com).

ABSTRACT This study introduces the development and application of Canonical polynomials as a novel alternative to Modified Laplace-Adomian Decomposition Method for solving nonlinear fractional partial differential equations. These polynomials are constructed to simplify complex computations by employing a recursive generation process, thereby reducing the overall computational complexity typically associated with such equations. To evaluate the effectiveness of the proposed approach, a selection of illustrative examples was analyzed. The results demonstrated excellent agreement with solutions reported in the existing literature, underscoring the method's accuracy and reliability. The recursive nature of Canonical polynomials not only enhances computational efficiency but also ensures a systematic and straightforward implementation process, making the method accessible for a wide range of applications. One of the most significant advantages of this approach is its adaptability for programming, as the recursive generation simplifies algorithmic design and execution. This feature makes the method particularly suitable for integration into modern computational tools and software packages. In addition to its computational merits, the proposed method's accuracy and efficiency establish it as a robust alternative for tackling nonlinear fractional partial differential equations. Its simplicity and versatility further contribute to its appeal, providing researchers and practitioners with a practical and reliable tool for addressing challenging problems in the field of fractional calculus. This study demonstrates that Canonical polynomials offer a promising pathway for advancing the solution techniques for nonlinear fractional equations, paving the way for further research and applications.

KEYWORDS: Accurate, Canonical polynomials, Effectiveness, Efficiency, Fractional partial difference equations, Modified Laplace Adomian decomposition method

I. INTRODUCTION

Fractional partial differential equations (FPDEs) have garnered significant attention in mathematics, sciences, social sciences, and engineering due to their wide-ranging applications in modeling complex physical and chemical processes. These equations are very useful for simulating things that happen in the real world, like viscoelasticity, electrochemical processes, strange diffusion, signal processing, and control theory. FPDEs are different from classical partial differential equations because they use fractional derivatives, which give a more accurate picture of memory and heritability in systems that are changing. Even though FPDEs are becoming more important, they are still very hard to solve, especially when the problems are not linear and there are not any analytical solutions. In Sultana, Waqar, Ali, Lupas, Ghanim and Abduljabar (2024), FPDEs are better suited for representing complex phenomena and process while (Noor, Alshehry, Khan and Khan, 2023) say the popularity of FPDEs increased as a result of their extraordinary capacity to offer more accurate simulations for a variety of physical phenomena.

Some of the methods used to solve FPDEs are Homotopy Analysis Method (Dehghan, Maanafian and Saadatmand, 2010), Laplace decomposition method (Khan, Hussain, Jafari and Khan, 2010), Modified Least Square Perturbation Method (Hayman and Subhash, 2018), Operational Matrix Method Based on Shifted Chebyshev Polynomials (Ali,

Abd El Salam, and Mohamed, 2019) and a Fast Numerical Method for FPDEs (Mockary, Baboliany and Vahidi, 2019). Some works on an iterative scheme have been focusing on the development of more advanced and efficient methods for solving FPDEs. These include the Variational Iterative Method (Wazwaz, 2011), Variational Method for Solving Linear and Nonlinear Systems of FPDEs (Wazwaz, 2007), Analytical Treatment of System of Linear and Nonlinear PDEs by Homotopy-Perturbation Method (Chowdhury, Hashim and Ismail, 2010) and to mention but few.

Other works include the application of Laplace-Adomian Decomposition Method for Nonlinear System of PDEs (Fadaei, 2011) and Fast Numerical Method for FPDEs (Mockary, Baboliany and Vahidi, 2019). The use of FPDEs is so immense that it has become an emerging area of current research because the equations appear in various applied fields. The existing drawbacks in most of the works of the researchers include that of (Khan, Shah. Kumam, Baleanu and Arif, 2020) which involve large matrices and required large computer memory, the method of Adomian decomposition involve construction of Adomian polynomials which is difficult to construct and cumbersome, the method of (Jafari, Nazari and Khalique, 2013) involved Laplace transform and hence, inverse Laplace transform in some works may not exist which made it encountered difficulty while some iterations are tedious and may converge at higher iterations. The primary objective of this study is to develop a robust and efficient approach for solving nonlinear FPDEs using the Modified Laplace-Adomian Decomposition Method. This approach aims to address the computational challenges associated with existing methods, such as the construction of Adomian Decomposition Method, the large computational memory required for handling complex matrices, and the difficulties posed by inverse Laplace transforms in certain cases. By refining these techniques, this study seeks to provide a more efficient and accessible solution framework for FPDEs. The motivation for this research lies in the increasing demand for accurate and efficient methods to model complex phenomena. As fractional partial differential equations (FPDEs) become integral to fields like engineering and the natural sciences, improving their solution techniques holds immense practical significance. The proposed method is expected to overcome some of the key limitations in existing approaches, offering a streamlined and effective alternative for researchers and practitioners. This study contributes to the growing body of knowledge on fractional partial differential equations (FPDEs) by presenting a novel computational framework that balances accuracy, efficiency, and simplicity, paving the way for further advancements in this dynamic research area.

II. DEFINITION OF RELEVANT TERMS

A. PARTIAL DIFFERENTIAL EQUATION

Partial differential equation is an equation which contains partial differential coefficients, independent variables and dependent variable (Dass, 2010).

B. LAPLACE TRANSFORM

Suppose $f(t)$ represents some positive values in t defined for $t \geq 0$, the Laplace transform of $f(t)$ is defined as (Dass and Verma, 2012):

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt \quad (1)$$

where s can be either real or complex.

C. FRACTIONAL DERIVATIVE OF $f(x)$ IN THE CAPUTO SENSE

The Caputo fractional derivative operator $D^v f(x)$ is defined as:

$$D^v f(x) = \frac{1}{\Gamma(m-v)} \int_0^x \frac{f^m(t)}{(x-t)^{v-m-1}} dt; x > 0 \quad (2)$$

where $m-1 < v < m, m \in \mathbb{N}$ (Mahdy and Mohamed, 2016), (Ali, Abd El Salam and Mohammed, 2019). In the Caputo derivative,

$$D^v x^n = \begin{cases} 0; n \in \mathbb{N}; n < [v] \\ \frac{\Gamma(n+1)}{(n+1-v)} x^{n-v}; n \in \mathbb{N}; n \geq [v] \end{cases} \quad (3)$$

The ceiling function $\lceil v \rceil$ denotes that the smallest integer greater than or equal to v while $\mathbb{N} = \{0, 1, 2, \dots\}$. Equally, the Caputo differential operator coincides with the usual differential operator of integer order.

D. LAPLACE TRANSFORM OF CAPUTO DERIVATIVE

According to Khan et al. (2020), this is defined as:

$$\mathcal{L}[D_*^\alpha f(t)] = s^\alpha F(s) - \sum_{k=0}^{m-1} s^{\alpha-1-k} f^{(k)}(0); m-1 < \alpha < m \quad (4)$$

E. LAPLACE DERIVATIVE OF ORDER n

Dass and Verma (2012) defined this as:

$$\mathcal{L}[f^n(t)] = s^n \mathcal{L}f(t) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s^{(n-1)} f^{(n-1)}(0) \quad (5)$$

F. GAMMA FUNCTION

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \quad (\text{Stroud and Booth, 2007}) \quad (6)$$

G. MITTAG-LEFFLER FUNCTION

According to Dhaigude, Kanade and Dhaigude (2018) and Jafari, Nazari and Khalique (2013), a parameter function commonly known as Mittag-Leffler function valid in the whole complex plane is defined by the series:

$$E_\alpha(z) = \sum_{k=0}^\infty \frac{z^k}{\Gamma(\alpha k + 1)}; \alpha > 0, z \in \mathbb{C} \quad (7)$$

H. CONSTRUCTION OF CANONICAL POLYNOMIALS

A single polynomials approximation of the unique solution $y(x)$ is of the form:

$$D^\alpha y(x) = f(x, y(x)); x > 0; a \leq x \leq b \quad (8)$$

with initial condition

$$y^k(0) = y(0); k = 0, 1, 2, \dots, (n-1), n \in \mathbb{N} \quad (9)$$

is determined in the form

$$y(x) \cong y_n(x) = \sum_{i=0}^N a_i Q_i(x); a \leq x \leq b \quad (10)$$

Here, $Q_i(x); i \leq 0$ are Canonical polynomials of degree N . Equation (8) can be re-written in the form:

$$D^\alpha y(x) + y(x) = f(x, y(x) + y(x)) \quad (11)$$

Now, let

$$L \equiv D^\alpha + 1 \quad (12)$$

Lanczos (1956) defines Canonical polynomials as:

$$L Q_i(x) = x^i; i = 0, 1, 2, \dots \quad (13)$$

From equation (12) and using the definition given in equation (3), it follows that

$$L x^i = \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} x^{i+\alpha} + x^i \quad (14)$$

Using the property of Caputo derivative in equation (14), it follows that

$$L x^i = \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} L Q_{i-\alpha}(x) + L Q_i(x) \quad (15)$$

From the linearity of \mathcal{L} and with the existence of \mathcal{L}^{-1} , equation (15) becomes:

$$\mathcal{L}^{-1}[L x^i] = \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} \mathcal{L}^{-1}[L Q_{i-\alpha}(x)] + \mathcal{L}^{-1}[L Q_i(x)] \quad (16)$$

$$Q_i(x) = x^i - \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} Q_{i-\alpha}(x) \quad (17)$$

Here, $Q_i(x)$ in equation (17) is further simplified using equation (13) thus:

$$Q_{i-\alpha}(x) = \mathcal{L}^{-1}x^{i-\alpha} \Rightarrow LQ_{i-\alpha}(x) = x^{i-\alpha} \quad (18)$$

$$LQ_{i-\alpha}(x) = x^{i-1+1-\alpha} = x^{i-1}x^{1-\alpha} = LQ_{i-1}(x)LQ_{1-\alpha}(x) \quad (19)$$

$$LQ_{i-\alpha}(x) = LQ_{i-1}(x)LQ_{1-\alpha}(x) \quad (20)$$

Introducing \mathcal{L}^{-1} into equation (20) and simplify to obtain:

$$\frac{Q_{i-\alpha}(x)}{Q_{i-1}(x)} = LQ_{i-\alpha}(x) \quad (21)$$

Equation (21) could be expressed as

$$Q_{i-\alpha}(x) = Q_{i-1}(x)LQ_{i-\alpha}(x) \quad (22)$$

Using equation (13) in equation (22),

$$Q_{i-\alpha}(x) = Q_{i-1}(x)x^{i-\alpha}(x) \quad (23)$$

Putting equation (23) in equation (17),

$$Q_i(x) = x^i - x^{i-\alpha} \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} Q_{i-1}(x) \quad (24)$$

From equation (24), the following set of Canonical polynomials are obtained:

$$\left. \begin{aligned} Q_0(x) &= 1 \\ Q_1(x) &= x - \frac{x^{1-\alpha}}{\Gamma(2-\alpha)} \\ Q_2(x) &= x^2 - \frac{2x^{2-\alpha}}{\Gamma(3-\alpha)} - \frac{2x^{-2+\alpha}}{(\Gamma(3-\alpha))^2} \\ Q_3(x) &= x^3 - \frac{6x^{3-\alpha}}{\Gamma(4-\alpha)} - \frac{12x^{3-2\alpha}(-3+\alpha)}{(\Gamma(4-\alpha))^2} + \frac{12x^{3-3\alpha}(-2+\alpha)(-3+\alpha)^2}{(\Gamma(4-\alpha))^3} \\ &\vdots \\ Q_N(x) &= x^N - \frac{x^{1-\alpha}\Gamma(N+1)}{\Gamma(N+1-\alpha)} Q_{N-1}(x); N > 0 \end{aligned} \right\} \quad (25)$$

Assumption

Here, the assumption is that for all $N \leq 0$, $Q_N(x) = 0$

III. METHODOLOGY

In this section, the Modified Laplace-Adomian Decomposition Method is used to solve system of nonlinear fractional partial differential equations. The modified is done by replacing the Adomian polynomials by the constructed Canonical polynomials. The system of nonlinear fractional partial differential equations considered is of the form given as:

$$\left. \begin{aligned} D_t^\alpha u(x, t) + L_1(u, v) + N_1(u, v) &= g_1(x, t) \\ D_t^\alpha u(x, t) + L_2(u, v) + N_2(u, v) &= g_2(x, t) \end{aligned} \right\} \quad (26)$$

together with initial conditions:

$$u(x, 0) = f_1(x, t); v(x, 0) = f_2(x, t) \quad (27)$$

Here, $D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$ is the Caputo fractional derivative of order α ; L_1, L_2 are linear operators; N_1, N_2 are nonlinear operators while g_1, g_2 are the source terms or given continuous functions.

Step 1

Applying the Laplace transform into equation (26) to obtain:

$$\left. \begin{aligned} \mathcal{L}[D_t^\alpha u(x, t)] + \mathcal{L}[L_1(u, v) + N_1(u, v)] &= \mathcal{L}[g_1(u, v)] \\ \mathcal{L}[D_t^\alpha u(x, t)] + \mathcal{L}[L_2(u, v) + N_2(u, v)] &= \mathcal{L}[g_2(u, v)] \end{aligned} \right\} \quad (28)$$

Step 2

Using the differentiation property of Laplace transform in (28) with the initial conditions in (27),

$$\left. \begin{aligned} \mathcal{L}[u(x, t)] &= \sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k u(x, t)}{\partial t^k} \Big|_{t=0} + \mathcal{L}[g_1(u, v)] - \mathcal{L}[L_1(u, v) - N_1(u, v)] \\ \mathcal{L}[v(x, t)] &= \sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k v(x, t)}{\partial t^k} \Big|_{t=0} + \mathcal{L}[g_1(u, v)] - \mathcal{L}[L_1(u, v) - N_1(u, v)] \end{aligned} \right\} \quad (29)$$

Step 3

The iteration formulae obtained in equation (29) are used to suggest the main iterative scheme that involves the Lagrange multiplier, $\lambda(s)$ defined as $\lambda(s) = \frac{1}{s^\alpha}$. Substituting for $\frac{1}{s^\alpha}$ in (29) to obtain:

$$\left. \begin{aligned} \mathcal{L}[u(x, t)] &= \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k u(x, t)}{\partial t^k} \Big|_{t=0} + \mathcal{L}[g_1(u, v)] - \mathcal{L}[L_1(u, v) - N_1(u, v)] \right] \\ \mathcal{L}[v(x, t)] &= \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k v(x, t)}{\partial t^k} \Big|_{t=0} + \mathcal{L}[g_1(u, v)] - \mathcal{L}[L_1(u, v) - N_1(u, v)] \right] \end{aligned} \right\} \quad (30)$$

The solutions of equation (30) are obtained using Modified Laplace-Adomian Decomposition Method (MLADM) defined by the infinite series of components given by:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t); \quad v(x, t) = \sum_{n=0}^{\infty} v_n(x, t) \quad (31)$$

The components u_n and v_n are determined recursively.

Since $N_i(u, v); i \geq 0$ are nonlinear, $N_i(u, v)$ can be written as $N(u, v) = uv$.

The Adomian polynomials are determined using:

$$N_{i\lambda}(Q_1, Q_2, Q_3, \dots, Q_n) = \sum_{n=0}^{\infty} A_{in} Q_1(\lambda); i = 1, 2, 3, \dots, n \quad (32)$$

Here, λ is a parameter introduced for convenience. Thus,

$$A_{in} = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} [N(Q_{1n}, Q_{2n}, \dots, Q_{nn})]; \quad (33)$$

$$\left. \begin{aligned} Q_{1i} &= \sum_{i=0}^n u_i Q_1(\lambda) \\ Q_{2i} &= \sum_{i=0}^n v_i Q_1(\lambda) \\ &\vdots \\ Q_{ni} &= \sum_{i=0}^n u_i Q_i(\lambda) \end{aligned} \right\} \quad (34)$$

when $n = 0$, in equation (35)

$$A_{i0} = \frac{1}{0!} \frac{\partial^0}{\partial \lambda^0} [N_0(Q_{10}, Q_{20}, \dots, Q_{n0})] \quad (36)$$

$$A_{i0} = u_0 Q_0(\lambda) = u_0 v_0 \quad (37)$$

Subsequently,

$$Q_{11} = \sum_{n=0}^1 u_i Q_i(\lambda) = u_0 Q_0(\lambda) + u_1 Q_1(\lambda) \quad (38)$$

$$Q_{21} = \sum_{n=0}^1 v_i Q_i(\lambda) = v_0 Q_0(\lambda) + v_1 Q_1(\lambda) \quad (39)$$

Hence,

$$A_{1i} = \frac{1}{1!} \frac{\partial}{\partial \lambda} \left[N_1 \left(u_0(1) + u_1 \left(\lambda - \frac{\lambda^{1-\alpha}}{\Gamma(2-\alpha)} \right), v_0(1) + v_1 \left(\lambda - \frac{\lambda^{1-\alpha}}{\Gamma(2-\alpha)} \right) \right) \right] \lambda = 0 \quad (40)$$

Simplifying equation (40),

$$A_{11} = u_0 v_1 + u_1 v_0 \quad (41)$$

$$A_{11} = \frac{1}{2!} \frac{\partial^2}{\partial \lambda^2} \left[N_2 \left(u_0(1) + u_1 \left(\lambda - \frac{\lambda^{1-\alpha}}{\Gamma(2-\alpha)} \right) + u_2 \left(\lambda^2 - \frac{2\lambda^{2-\alpha}}{\Gamma(3-\alpha)} \right) - \frac{2\lambda^{2+\alpha}}{\Gamma(3-\alpha)} \left(v_0(1) + v_1 \left(\lambda - \frac{\lambda^{1-\alpha}}{\Gamma(2-\alpha)} \right) \right) + \left(\lambda^2 - \frac{2\lambda^{2-\alpha}}{\Gamma(3-\alpha)} \right) - \frac{2\lambda^{2+\alpha}}{(\Gamma(3-\alpha))^2} \right] \lambda = 0 \quad (42)$$

Hence,

$$A_{21} = u_0 v_2 + u_1 v_1 + u_2 v_0 \quad (43)$$

From equations (36), (40), (42), ...

$$\left. \begin{aligned} A_{i0} &= u_0 v_0 \\ A_{i1} &= u_0 v_1 + u_1 v_0 \\ A_{i2} &= u_0 v_2 + u_1 v_1 + u_2 v_0 \\ A_{i3} &= u_3 v_0 + u_2 v_1 + u_1 v_2 + u_0 v_3 \\ &\vdots \end{aligned} \right\} \quad (44)$$

Step 4

Substituting equation (42) into the Laplace transform given in equation (30) to obtain

$$\mathcal{L}[\sum_{k=0}^n u(x, t)] = \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k u(x, t)}{\partial t^k} \Big|_{t=0} \right] + \frac{1}{s^\alpha} [g_1(x, t)] - \frac{1}{s^\alpha} [L_1(\sum_{k=0}^n u(x, t), \sum_{k=0}^n v(x, t)) - \sum_{k=0}^n A_{0j}] \quad (45)$$

$$\mathcal{L}[\sum_{k=0}^n v(x, t)] = \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k v(x, t)}{\partial t^k} \Big|_{t=0} \right] + \frac{1}{s^\alpha} [g_1(x, t)] - \frac{1}{s^\alpha} [L_1(\sum_{k=0}^n u(x, t), \sum_{k=0}^n v(x, t)) - \sum_{k=0}^n A_{0j}] \quad (46)$$

Applying the linearity of the Laplace transform in equations (45) and (46), the following are defined recursively:

$$\left. \begin{aligned} \mathcal{L}[u_0(x, t)] &= \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k u(x, t)}{\partial t^k} \Big|_{t=0} \right] + \frac{1}{s^\alpha} [g_1(x, t)] \\ \mathcal{L}[u_1(x, t)] &= -\frac{1}{s^\alpha} [N_1(u_0(x, t), v_0(x, t))] - \frac{1}{s^\alpha} [A_0] \\ &\vdots \\ \mathcal{L}[u_{k+1}(x, t)] &= -\frac{1}{s^\alpha} [N_k(u_k(x, t), v_k(x, t))] - \frac{1}{s^\alpha} [A_k] \end{aligned} \right\} \quad (47)$$

$$\left. \begin{aligned} \mathcal{L}[v_0(x, t)] &= \frac{1}{s^\alpha} \left[\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k v(x, t)}{\partial t^k} \Big|_{t=0} \right] + \frac{1}{s^\alpha} [g_1(x, t)] \\ \mathcal{L}[v_1(x, t)] &= -\frac{1}{s^\alpha} [N_1(u_0(x, t), v_0(x, t))] - \frac{1}{s^\alpha} [A_{0j}] \\ &\vdots \\ \mathcal{L}[v_{k+1}(x, t)] &= -\frac{1}{s^\alpha} [N_n(u_k(x, t), v_k(x, t))] - \frac{1}{s^\alpha} [A_{nj}] \end{aligned} \right\} \quad (48)$$

Step 5

Taking the inverse Laplace transforms of equations (47) and (48), u_k and v_k are determined where

IV. NUMERICAL APPLICATION

In this section, MLADM is used to solve nonlinear fractional partial differential equations.

Example 4.1

Consider the initial value problem for nonlinear system of fractional Burger's equations:

$$\left. \begin{aligned} D_t^\alpha u - D_x^2 u + u D_x u - uv &= 0 \\ D_t^\alpha v - D_x^2 v + v D_x v + uv &= 0; 0 < \alpha \leq 1 \end{aligned} \right\} \quad (49)$$

with initial conditions

$$u(x, 0) = \sin x; u(x, 0) = \cos x \quad (50)$$

where the exact solution is $\alpha = 1$.

Source: (Dhaigude *et al.*, 2018)

Example 4.2

Consider the initial value problem for nonlinear system fractional Burger's equation:

$$\left. \begin{aligned} D_t^\alpha u + D_x^2 u + 2u D_x u + D_x(uv) &= 0 \\ D_t^\alpha v + D_x^2 v + 2v D_x v + D_x(uv) &= 0; 0 < \alpha \leq 1 \end{aligned} \right\} \quad (51)$$

with initial conditions:

$$u(x, 0) = e^x; v(x, 0) = -e^x \quad (52)$$

Source: (Fadaei, 2011).

Example 4.3

Solve the inhomogeneous nonlinear system of fractional partial differential equations:

$$\left. \begin{aligned} D_t^\alpha u(x, t) + v(x, t)u_x(x, t) + u(x, t) &= 1 \\ D_t^\alpha v(x, t) - u(x, t)v_x(x, t) - v(x, t) &= 1; 0 < \alpha \leq 1 \end{aligned} \right\} \quad (53)$$

with initial conditions:

$$u(x, 0) = e^x, v(x, 0) = e^{-x} \quad (54)$$

and exact solution is $\alpha = 1$.

Source: (Fadaei, 2011, Chowdhury *et al.*, 2010, Wazwaz, 2007).

V. RESULTS AND DISCUSSION

Table 1: Comparison of modified Laplace-Adomian decomposition method with the exact solution of $u(x, t)$ for equation (49) for $t = 0.01, 0.05, 0.75$ and $0.10; -2 \leq x \leq 2$ and $\alpha = 1.00$.

t	x	$\alpha = 1.00$	$\alpha = 1.00$	$\alpha = 1.00$
		Exact	MLADM	Error = Exact - MLADM
0.01	-2.0	0.019900325	0.019900325	0.000000000
	-1.0	0.009950166	0.009950166	0.000000000
	0.0	0.000000000	0.000000000	0.000000000
	1.0	-0.009950166	-0.009950166	0.000000000
	2.0	-0.019900332	-0.019900332	0.000000000
0.05	-2.0	0.097541146	0.097541146	0.000000000
	-1.0	0.048770573	0.048770573	0.000000000
	0.0	0.000000000	0.000000000	0.000000000
	1.0	-0.048770573	-0.048770573	0.000000000
	2.0	-0.097541146	-0.097541146	0.000000000
0.10	-2.0	0.190325000	0.190325000	0.000000000
	-1.0	0.009516250	0.009516250	0.000000000
	0.0	0.000000000	0.000000000	0.000000000
	0.1	-0.09516250	-0.09516250	0.000000000
	2.0	-0.190325000	-0.190325000	0.000000000

In table 1, the results obtained using the Modified Laplace-Adomian Decomposition Method (MLADM) are in excellent agreement with the exact solutions of $u(x, t)$ to 9 decimal places.

Table 2: Comparison of Approximate Solution of $u(x, t)$ for Equation (49) for $t = 0.01, 0.05$, and $0.10; -2 \leq x \leq 2$ and $\alpha = 0.25, 0.50, 0.75, 1.00$ between MLADM and Dhaigude *et al.* (2018)

t	x	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1.00$
		$u_{MLADM} = \text{Dhaig 2018}$	$u_{MLADM} = \text{Dhaig 2018}$	$u_{MLADM} = \text{Dhaig 2018}$	$u_{MLADM} = \text{Dhaig 2018}$
0.01	-2.0	0.520904022	0.207080339	0.067335259	0.019900325
	-1.0	0.260452011	0.103540169	0.0336676300	0.009950166
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	1.0	-0.433270425	-0.243015505	-0.009950166	-0.811430888
	2.0	-0.866540851	-0.486031010	-0.228617756	-0.019900332

0.05	-2.0	0.668872676	0.418947388	0.214162704	0.097541146
	-1.0	0.334436338	0.209473694	0.107081352	0.048770573
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	1.0	-0.334436338	-0.209473694	-0.107081352	-0.048770573
	2.0	-0.668872677	-0.418947388	-0.214162704	-0.097541146
0.1	-2.0	0.714147523	0.551226290	0.343478636	0.190325000
	-1.0	0.357073761	0.275613145	0.171739318	0.009516250
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	0.1	-0.357073761	-0.275613145	-0.171739318	-0.09516250
	2.0	-0.714147523	-0.551226290	-0.343478636	-0.190325000

In Table 2, it is observed that the approximate solutions of $u(x, t)$ obtained in MLADM and the approximate solution of Dhaigude (2018) are in good agreement at the given values x and to 9 decimal places. The approximate solutions converge to the exact solution at the said given values t, x , and α .

Table 3: Comparison of Approximate Solutions of $v(x, t)$ of equation (49) for $t = 0.01, 0.05$, and 0.1 ; $-2 \leq x \leq 2$; $\alpha = 0.25, 0.50, 0.75, 1.00$ between MLADM and Dhaigude *et al.* (2018).

t	x	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1.00$
		$u_{MLADM} = Fad\ 2011$	$u_{MLADM} = Fad\ 2011$	$u_{MLADM} = Fad\ 2011$	$u_{MLADM} = Fad\ 2011$
0.01	-2.0	0.560904022	0.207080339	0.067335926	0.199003341
	-1.0	0.280452011	0.103540170	0.033667963	0.009950167
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	1.0	-0.280452011	-0.103540170	-0.033667963	-0.009950167
	2.0	-0.560904022	-0.207080339	-0.067335926	-0.199003341
0.05	-2.0	0.868872677	0.418947388	0.214246037	0.097542188
	-1.0	0.434436338	0.209473694	0.107123019	0.048771094
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	1.0	-0.434436338	-0.209473694	-0.107123019	-0.048771094
	2.0	-0.868872677	-0.418947388	-0.214246037	-0.097542188
0.1	-2.0	1.114147523	0.551226290	0.344145303	0.190341667
	-1.0	0.557073761	0.275613145	0.172072651	0.095170833
	0.0	0.000000000	0.000000000	0.000000000	0.000000000
	0.1	-0.557073761	-0.275613145	-0.172072651	-0.095170833
	2.0	-1.114147523	-0.551226290	-0.344145303	-0.190341667

Table 3 shows the approximate solutions of $v(x, t)$ for equation (49). When the results are compared with the results of Dhaigude *et al.* (2018), it is observed that the approximate solutions for both MLADM and Dhaigude *et al.* (2018) are in good agreement with the exact solution to 9 decimal places; the results converge to the exact solutions

Table 4: Comparison of Exact Solutions with Modified Laplace-Adomian Decomposition

Method for $u(x, t)$ in equation (70) for $t = 0.01, 0.05, 0.10$; $-2 \leq x \leq 2$ and $\alpha = 1.0$.

t	x	$\alpha = 1.0$	$\alpha = 1.0$	$\alpha = 1.0$
		u_{Exact}	u_{MLADM}	Error
0.01	-2.0	-0.001346608	-0.001346608	0.000000000
	-1.0	-0.003660462	-0.003660462	0.000000000
	0.0	-0.009950166	-0.009950166	0.000000000
	1.0	-0.027047356	-0.027047356	0.000000000
	2.0	-0.073522336	-0.073522336	0.000000000
0.05	-2.0	-0.063829719	-0.063829719	0.000000000
	-1.0	-0.017941691	-0.017941691	0.000000000
	0.0	-0.048770573	-0.048770573	0.000000000
	1.0	-0.132572162	-0.132572162	0.000000000
	2.0	-3.484984596	-3.484984596	0.000000000
0.1	-2.0	-0.012878844	-0.012878844	0.000000000
	-1.0	-0.035008327	-0.035008327	0.000000000
	0.0	-0.095162500	-0.095162500	0.000000000
	0.1	-0.258678494	-0.258678494	0.000000000
	2.0	-0.703161051	-0.703161051	0.000000000

In Table 4, the results of the MLADM for $u(x, t)$ in equation (70) for $t = 0.01, 0.05$ and 0.10 ; $-2 \leq x \leq 2 = 1.00$ show that the solutions are in agreement with the exact solutions. It is also observed that the results converge to the exact solutions.

Table 5: Comparison of Approximate Solutions of $u(x, t)$ for equation (70) for values of $t = 0.01, 0.05, 0.10$; $-2 \leq x \leq 2$; $\alpha = 0.25, 0.50, 0.75$ and 1.00 between MLADM and Fadaei (2011).

t	x	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1 (Exact)$
		$UMALAD = Fadaei\ 2011$	$UMALAD = Fadaei\ 2011$	$UMALAD = Fadaei\ 2011$	$UMALAD = Fadaei\ 2011$

0.01	-2.0	-0.035248347	-0.0140126382	-0.004556418	-0.001346608
	-1.0	-0.095814940	-0.038090300	-0.012385629	-0.003660462
	0.0	-0.260450111	-0.103540170	-0.033667630	-0.009950166
	1.0	-0.707981969	-0.281451361	-0.091518106	-0.027047356
	2.0	-1.924494521	-0.765064120	-0.248772004	-0.073522336
0.05	-2.0	-0.045261037	-0.028349182	-0.014491885	-0.006382972
	-1.0	-0.123032253	-0.077061065	-0.039393028	-0.017941691
	0.0	-0.334436338	-0.280604299	-0.107081352	-0.048770573
	1.0	-0.909092221	-0.569408535	-0.291077293	-0.132572162
	2.0	-2.471168863	-1.547812875	-0.791230116	-0.484984596
0.1	-2.0	-0.048324679	-0.037300183	-0.023242389	-0.012878844
	-1.0	-0.131360096	-0.101392410	-0.063240616	-0.035008327
	0.0	-0.370737614	-0.275613145	-0.171739318	-0.095162500
	0.1	-0.970627116	-0.749194203	-0.466835868	-0.258678494
	2.0	-2.638438052	-2.236676283	-1.268991455	-0.703161051

In Table 5, the results of the MLADM for $u(x, t)$ in equation (70) for $t = 0.01, 0.05$ and 0.10 ; $-2 \leq x \leq 2$, $\alpha = 0.25, 0.50, 0.75$ and 1.00 are compared with the works of Fadaei (2011). The results are in good agreement with the exact solutions, e^{-x} to 9 decimal places and converge to the exact solutions.

Table 6: Comparison of Approximate Solutions of $u(x, t)$ for equation (70) for values of $t = 0.01, 0.05, 0.10$; $-2 \leq x \leq 2$; $\alpha = 0.25, 0.50, 0.75$ and 1.00 between MLADM and Fadaei (2011).

t	x	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$ (Exact)
		UMALAD = Fadaei 2011	UMALAD = Fadaei 2011	UMALAD = Fadaei 2011	UMALAD = Fadaei 2011
0.01	-2.0	-0.035248347	-0.0140126382	-0.004556418	-0.001346608
	-1.0	-0.095814940	-0.038090300	-0.012385629	-0.003660462
	0.0	-0.260450111	-0.103540170	-0.033667630	-0.009950166
	1.0	-0.707981969	-0.281451361	-0.091518106	-0.027047356
	2.0	-1.924494521	-0.765064120	-0.248772004	-0.073522336
0.05	-2.0	-0.045261037	-0.028349182	-0.014491885	-0.006382972
	-1.0	-0.123032253	-0.077061065	-0.039393028	-0.017941691
	0.0	-0.334436338	-0.280604299	-0.107081352	-0.048770573
	1.0	-0.909092221	-0.569408535	-0.291077293	-0.132572162
	2.0	-2.471168863	-1.547812875	-0.791230116	-0.484984596
0.1	-2.0	-0.048324679	-0.037300183	-0.023242389	-0.012878844
	-1.0	-0.131360096	-0.101392410	-0.063240616	-0.035008327
	0.0	-0.370737614	-0.275613145	-0.171739318	-0.095162500
	0.1	-0.970627116	-0.749194203	-0.466835868	-0.258678494
	2.0	-2.638438052	-2.236676283	-1.268991455	-0.703161051

Table 6 is the approximate solution of $v(x, t)$ for equation (70) using MLADM for values of $t = 0.01, 0.05$, and 0.10 , $\alpha = 0.25, 0.50, 0.75$ and 1.00 . It is observed that the approximate solutions

VI. CONCLUSION

In this work, MLADM has been effectively and efficiently applied to solve nonlinear systems of FPDEs. The method has been successfully employed to obtain the analytical approximate solution of the systems. The calculations that are involved in the method are straight forward and simple to apply. Also, the solutions are provided in terms of convergent series whose components could easily be computed. It could be implemented using the MAPPLE 18 software. The method also maintains a high level of accuracy of the numerical results. The method can be considered to be a nice refinement in comparison with the existing numerical techniques. Where there are nonlinear fractional partial differential equations that the exact solutions do not exist, a few numbers of approximations should be considered for numerical purposes. Other methods could be considered for equations like time-dependent equations and evolution equations for future studies.

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