

Portfolio Optimization Technique: Mean – Variance and Bayesian Approaches in the Nigerian Stock Market

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ABSTRACT

The Nigerian Stock Market provides a complex and dynamic environment for investors. In this dynamic context, portfolios optimization is mandatory for an effective risk-return balance with the objective of accomplishing financial goals and objectives. This study delves into two prominent portfolio optimization methodologies: the Mean-Variance Approach and the Bayesian Approach. The mean-variance approach, sought to solve the optimization problem by analyzing the means and variances of a certain collection of stocks in order to achieve maximum return on investment. However, the simplicity of the mean-variance approach gives rise to various limitations. This study seeks to address some of these limitations by employing the Bayesian method, which is primarily grounded in probability theory and Bayes' theorem. The methodologies were used to construct optimal portfolios with daily data of the Nigerian stock market from 2018 to 2024 with particular focus on the NSE 30 equities of the Nigerian stock market. The work is based on building optimizations models, a statistical analysis of the historical market data and a review of recent data, effects and market expectations. The results of this study demonstrate the flexibility of Bayesian portfolios to construct portfolios that potentially produce higher returns by various weightings of past and recent data. However, the mean-variance approach is good in the matter of risk-return balance. The paper contributes to the current discourse on the effective portfolio optimization strategies by providing an in-depth knowledge of their implications in the Nigerian Stock Market.

KEYWORDS: Optimization, Portfolio, Statistical Analysis, Risk and Returns

I. INTRODUCTION

A portfolio is the number of stocks, bonds or other financial or real assets that an individual, group of people or a company owns with the goal of making profits. Portfolio optimization defines an outline for the construction of diversified portfolios that are optimized for the best possible risk-return ratio. The Nigerian Stock Market is a vibrant nucleus of economic activity in the dynamic West African environment, reflecting the potential opportunities and challenges associated with investments in emerging markets. The Nigerian Stock Market is an essential component of the Nigerian economy that contributes significantly to capital formation, investment opportunities, and economic development (Adewole, 2024; Ibadin & Egwaikhide, 2016). In a financial world of constant flux, investors are confronted with the critical task of assembling portfolios that leverage the potential for returns, while also traversing the complex landscape of market uncertainties. Portfolio optimization, a field rich with established and innovative techniques. Traditionally, portfolio development has been driven by the Mean-Variance Optimization (MVO), a concept

proposed by Harry Markowitz in the mid-20th century. However, the Nigerian Stock Market, distinguished by its particular quirks and socio-economic dynamics, provides a challenge to the long-standing conventional wisdom widespread in financial literature. Standing at the intersection of tradition and innovation, there is a clear urgency to assess and change approaches for optimizing portfolios. While the economic resiliency and development potential of Nigeria entice investors, the inherent volatility and unpredictability of financial markets need a nuanced approach to portfolio management. Bayesian Approach, characterized by its ability to integrate subjective views and adapt to dynamic market conditions, emerges as an innovative beacon, potentially giving a more robust framework for navigating the convoluted web of risks and returns. Bayesian Optimization is a more general optimization technique applicable across numerous domains. It adaptively balances exploration and exploitation and also suitable for uncertainties and complexities in the objective function (Shahriari et al., 2016).

II. RELATED WORKS

The area of portfolio optimization has been extensively covered in the existing literature on global financial markets but studies that focus on the specificities of the Nigerian Stock Market are rather sparse. Therefore, very few research papers have been published on portfolio estimation methods on the Nigerian landscape (Nigeria Stock Exchange). Nnanwa et al. 2016 evaluated the Mean-Variance method using historical data of listed financial companies on the Nigerian Stock Exchange (NSE) from 2009 to 2015, evaluated risk-return profiles and contrasted the efficient frontier portfolios of the model with the actual market performance. The effectiveness of the Mean-Variance approach in constructing efficient portfolios with optimal risk-return trade-offs was established. Moreover, Adewuyi & Ogundimu (2010) used the mean-variance approach in the Nigerian context and showed its effectiveness in achieving efficient frontiers. Limitations, however, include sensitivity to estimation errors and assumptions of normality. Also, Ofikwu (2017) conducted an analysis to compare the Mean-Variance approach with the Single Index Model using historical data of the 10 most capitalized companies on the Nigeria Stock Exchange from 2010 to 2016. Assessed portfolio performance on the basis of risk-return measures and tracking error. The results showed that the Single Index Model slightly outperformed the Mean-Variance approach in terms of risk adjusted returns and tracking error. Offiong et al. (2016) also employed the mean-variance method to determine the optimal portfolio in a three-asset portfolio mix in Nigeria. The study found that the only efficient optimal assets in the mix were Guinness and First Bank, which gave high returns at minimum variance. Ditimi et al. (2018) in Nigeria used time series analysis to show the nexus between macroeconomic fundamentals and stock prices. The study found a positive relationship between macroeconomic variables and stock prices and recommended policy interventions to stabilize the exchange rate and build the mindset of the institutional investors in the Nigerian stock market. Abdullahi et al. (2019) investigated the effect of market liquidity, inflation and exchange rates on stock return in the Nigerian Stock Exchange market. Market liquidity, exchange and inflation rates are found to influence stock return and demutualization and transparency in the exchange rate policies are recommended. Yang et al. (2013) proposed a forecasting-mean-correlation-entropy portfolio optimization model using fuzzy time series techniques as well as entropy as a risk measurement. The study compared the proposed model with conventional portfolio models and proved that the proposed model was an effective tool in portfolio selection. Kumar et al. (2022) applied the Markowitz portfolio optimization technique based on mean- variance and semi-variance as measures of risk on stocks listed on the

South Pacific Stock Exchange, Fiji. It was discovered that well-diversified portfolios (market portfolio and minimum- variance portfolio) with short-selling constraints have relatively higher expected returns with lower risk. Moreover, well-diversified portfolios perform better than the naïve and maximum portfolios in terms of risk.

Reviews of literature revealed that the mean-variance approach has been widely used in portfolio optimization in Nigerian stock markets. Actually, the portfolio optimization method has matured considerably (Jayeola & Olatunji (2025); Okonkwo et al (2024); Okafor & Robertson (2022); AlHalaseh & Shawawreh (2024); Izza et al (2024); Supandi & Oktavia (2026)). However, application of alternative techniques such as the Bayesian approach has not been extensively investigated in the Nigerian market space. Bayesian optimization approach is a powerful alternative that uses prior information and statistical techniques. (Nguyen, 2019) extended his research on portfolio optimization to a market outside of Nigeria by comparing the mean-variance approach and the Bayesian approach for 27 Dow Jones Companies from 2008-2017. The Bayesian approach was found to increase the accuracy of the parameter's estimation of the stocks and to reduce the amount of information needed to build the optimal portfolio. Schervish and Tsay, (1988) analyses Bayesian portfolio selection with conjugate priors and demonstrate that it can outperform classical methods in small-sample settings. Mironov (2026) conducts a comparative analysis of portfolio optimization methods with a focus on Bayesian approaches, applying them to a dataset of AI-related stocks from the U.S. market. The performance was evaluated in terms of risk-adjusted returns, particularly the Sharpe ratio, demonstrating the potential advantages of Bayesian optimization in fast-evolving sectors like artificial intelligence. The research finds that although the Markowitz model achieved the highest Sharpe ratio but also involved the highest concentration risk. He discovered that the more advanced the Bayesian model, the higher the Sharpe ratio. Chen et al. (2019), Jiang & Liu (2026) employed Bayesian approaches in portfolio optimization, revealing their robustness in handling complex financial scenarios. Bodnar et al. (2025) introduces Bayesian inference procedures for tangency portfolio with a primary focus on deriving a new conjugate prior for portfolio weights, the results revealed the performance of their leverage conjugate priors over the existing trading strategy. The purpose of this study is to evaluate and compare the efficiency of portfolio optimization strategies, specifically the Mean-Variance Approach (MVO) and the Bayesian Approach, in the distinctive context of the Nigerian Stock Market. This study seeks to address these essential difficulties by investigating the effectiveness of the Mean-Variance Approach in building optimal portfolios within the Nigerian Stock Market's specific barriers. Furthermore, it aims to examine the potential benefits and challenges of utilizing the Bayesian Approach, which is known for its flexibility to adapt to subjective perspectives and changing market conditions. This research aims to identify and analyze these problems as this would provide valuable insights to the on-going discourse on portfolio optimization and enhance the applicability of such methodologies in the Nigerian financial sector.

III. METHODOLOGY

This research studies the performance of the Mean-Variance (MV) and Bayesian methods of portfolio optimization in the Nigerian stock market.

A. Data Collection Procedure

The first phase of this study entails extensive data acquisition from pertinent sources within the Nigerian Stock Market. They include; stock prices, returns, and other financial metrics data 2018-2024, obtained through the S&P Capital IQ. The data comprises the daily closing stock prices for the above-mentioned time duration, data on risk-free rates and other macro-economic factors that may impact the stock market.

B. Mean-Variance Optimization (MVO)

Mean-Variance Optimization (MVO) revolutionized portfolio theory by providing investors with a systematic way to control risk and return. Mean-Variance Optimization is the basis of modern portfolio theory and was first proposed by Harry Markowitz in the 1950's. The main purpose is to optimize the asset allocation of a portfolio in order to minimize the total risk of the portfolio, as expressed by the variance or standard deviation of returns, and to maximize the expected returns. The MVO is computed on the following subset;

Stock return is the benefit an investor gets for having stock either in the form of dividends or capital gain. It is driven by factors such as market sentiment, economic conditions and company performance. In order to predict future performance, return can be computed historically using historical stock prices. The return of stock s at time t can be calculated as follows:

$$R_{st} = \frac{P_{st} - P_{s(t-1)}}{P_{s(t-1)}} \quad (1)$$

where the closing price of stock s at time t is P_{st} and at time $(t - 1)$ is $P_{s(t-1)}$. The expected return is found by averaging these historical returns over a number of time periods:

$$E(R_{st}) = \frac{\sum_{s=1}^n R_{st}}{n} \quad (2)$$

where n signifies the number of observed periods.

Besides rewards, investors should also take into account the risk of investing in shares, it is the uncertainty of getting the expected returns. Risk may be unsystematic, that is, specific to individual firms, or systematic, arising from macro-economic factors. There is generally more risk with a greater expected return. One way to measure this risk is to calculate the variance, this measures how much the returns vary from the expected value:

$$\sigma_s^2 = \sum_{t=1}^n \frac{(R_{st} - E(R_{st}))^2}{n-1} \quad (3)$$

However, the relationship or co-movement between two stock returns can be evaluated through covariance:

$$\sigma_{sk} = \sum_{t=1}^n \frac{((R_{st} - E(R_{st})) \cdot (R_{kt} - E(R_{kt})))}{n} \quad (4)$$

These risk metrics are important to assess the performance of the portfolio and to design an optimal mix of assets.

C. Portfolio

Portfolio a collection of assets owned by investor. This is usually done by diversification, which includes the spread of investments across different businesses or financial instruments. The primary goal of portfolio building is to minimize investment risk; by spreading capital over multiple choices, investors avoid the potential of losing a lot of money by investing everything

into one asset that may not do well. Diversification allows investors to deal with the uncertainty of market movements, by offsetting the possible losses of one investment against the possible gains of another (Sukor & Halim, 2023). Let ε denote the vector of expected returns, If ω is the vector of portfolio weights and ω is a unit vector, as in Vimelia et al. (2024). They are defined as follows:

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{pmatrix} \quad (5)$$

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_m \end{pmatrix} \quad (6)$$

$$\epsilon = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad (7)$$

The portfolio returns R_p , assuming $\sum_{s=1}^M \omega_s = 1$ expressed as;

$$R_p = \sum_{s=1}^M \sum_{t=1}^N \omega_i r_{s,t} = \omega^T r \quad (8)$$

The expected portfolio return ε_p can be defined as:

$$\varepsilon_p = E(r_p) = \sum_{s=1}^M \omega_s E(r_{s,t}) = \omega^T \varepsilon + \varepsilon^T \omega \quad (9)$$

The correlation (ρ_{ij}) between two assets, i and j , is determined as follows:

$$\rho_{ij} = \frac{cov_{ij}}{\sigma_i \sigma_j} \quad (10)$$

where σ_i and σ_j are the standard deviations of assets i and j respectively.

D. Variance of a Portfolio

Markowitz derived the formula for the variance of a n-asset portfolio as follows:

$$\sigma_p = \sum_{i=1}^n \sum_{j=1}^n w_i w_j cov_{ij} \quad (11)$$

$$\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j cov_{ij} = w \Sigma w^T \quad (12)$$

where: w_i is the weight of asset i in the portfolio

σ_i^2 is the variance of asset i in the portfolio

Cov_{ij} is the covariance between asset i and asset j in the portfolio

Σ is the covariance matrix of the n-asset portfolio

$W = [w_1, w_2, w_3, \dots, w_n]$ is the $1 \times n$ weight vector of the portfolio

w^T is the transposition of w

E. The Bayesian Method in Finding Posterior Distribution

Applying the Bayes' formula to continuous random variable θ with known prior distribution and a sample data set X as new information, we can obtain the posterior distribution of θ by rewriting the Bayes' theorem:

$$f(X|\theta) = \frac{f(X|\theta)f(\theta)}{f(X)} \quad (13)$$

$$= \frac{L(\theta|X)f(\theta)}{f(X)} \quad (14)$$

Where $(\theta|X)$ denotes the density function for the posterior distribution of θ

$(\theta|X)f(x, \theta)$ is the likelihood function of θ given data set X

(θ) is the prior distribution of the unknown parameter θ

$f(X)$ is the unconditional probability of the data set X

Since $f(X)$ does not depends on θ , equation can be rewritten as:

$$f(\theta|X) \propto L(\theta|X)f(\theta) \quad (15)$$

F. Bayesian Optimization

Bayesian methods are becoming more popular for portfolio optimization because of their ability in handling uncertainty and incorporate subjective information. Bayesian optimization means changing probability distributions in response to new data. Bayesian optimization is an advanced optimization method that utilizes probabilistic models to logically investigate and utilize the parameter space, aiming to identify the optimal configuration within a specified domain. The method combines a probabilistic surrogate model, which is usually modelled as a Gaussian process, with a probabilistic acquisition function. Bayesian Portfolio Optimization combines historical data (prior) with new market evidence (Likelihood to estimate updated return and risk measures i. e. the posterior. The prior for the research is an informative conjugate prior. This study focused on evaluating the efficacy of the conjugate prior, the mean vector of returns was assigned a k-dimensional multivariate normal prior conditioned on Σ while Σ was assigned an inverse Wishart prior. The covariance matrix (Σ) is updated by combining a prior distribution representing initial beliefs with a likelihood function of new observed asset returns. This process updates the expected returns and covariance matrices to yield robust asset weights. The specification of the prior distribution is expressed below:

$$\mu|\Sigma \sim N(\mu_p, \frac{1}{k_p} \Sigma) \quad (16)$$

The specification of Σ distribution is expressed below;

$$\Sigma \sim \text{INW}(v_p, \vartheta_p) \quad (17)$$

Where μ_p represents the investor's subjective belief or prior estimate of the expected return for each asset, and k_p illustrates the prior precision, indicating the confidence level in the prior belief on μ_p . $\text{INW}(v_p, \vartheta_p)$ signifies inverse Wishart distribution with v_p degrees of freedom and parameter matrix ϑ_p . Considering set of daily returns data that follows normal distribution, the likelihood function is given by;

$$L(Y|\mu, \Sigma) = L(y_1, \dots, y_n|\mu, \Sigma) = \prod_{i=1}^n (2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (y_i - \mu)^T \Sigma^{-1} (y_i - \mu) \right\} \propto |\Sigma|^{-\frac{n}{2}} \exp \left\{ -\frac{1}{2} \sum_{i=1}^n (y_i - \mu)^T \Sigma^{-1} (y_i - \mu) \right\} \quad (18)$$

By expression of Bayes theorem above, the posterior of μ is:

$$P(\mu|\Sigma, Y) \propto L(Y|\mu, \Sigma) P(\mu|\Sigma) \quad (19)$$

Expanding equation (19), Prior of μ is expressed below as;

$$(2\pi)^{-\frac{k}{2}} \left| \frac{1}{k_p} \Sigma \right|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mu - \mu_p)^T \frac{1}{k_p} \Sigma^{-1} (\mu - \mu_p) \right\} \quad (20)$$

Let $\frac{1}{k_p} \Sigma$ be represented by M_p , then the Prior of μ can be rewritten as;

$$P(\mu) = |M_p|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mu^T M_p^{-1} \mu + 2(\mu^T M_p^{-1} \mu_p - 2\mu^T M_p^{-1} \mu_p)) \right\} \quad (21)$$

However, $\mu^T M_p^{-1} \mu_p$ is a constant, it leads equation (21) to be rewritten as;

$$P(\mu) = |M_p|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mu^T M_p^{-1} \mu - 2\mu^T M_p^{-1} \mu_p) \right\} \quad (22)$$

$$P(\mu) = |M_p|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mu^T B_p \mu + \mu^T b_p) \right\} \quad (23)$$

where B_p signifies M_p^{-1} and b_p denotes $M_p^{-1} \mu_p$.

Posterior of μ is expressed below in line with equation (15)

$$P(\mu|\Sigma, Y) \propto |\Sigma|^{-\frac{n}{2}} \exp \left\{ -\frac{1}{2} \mu^T n \Sigma^{-1} \mu + \mu^T n \Sigma^{-1} Y \right\} * |M_p|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mu^T B_p \mu + \mu^T b_p) \right\} \quad (24)$$

$$P(\Sigma|Y) \propto L(\Sigma|Y) P(\Sigma) \quad (25)$$

$$L(\Sigma|Y) = \prod_{i=1}^n (2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (y_i - \mu)^T \Sigma^{-1} (y_i - \mu) \right\} \quad (26)$$

Hence;

$$-\frac{1}{2} \sum_{i=1}^n (y_i - \mu)^T \Sigma^{-1} (y_i - \mu) = -\frac{1}{2} \text{tr } S \Sigma^{-1} \quad (27)$$

Where:

$$S = \sum_i ((y_i - \bar{y})(y_i - \bar{y})^T) \quad (28)$$

Equation (26) becomes:

$$L(\Sigma|Y) = |\Sigma|^{-\frac{n}{2}} \exp -\frac{1}{2} \text{tr } S \Sigma^{-1} \quad (29)$$

Considering equation (25), the posterior of the covariance Σ is expressed as

$$P(\Sigma|Y) \propto |\Sigma|^{-\frac{n}{2}} \exp \left\{ -\frac{1}{2} \text{tr } S \Sigma^{-1} \right\} * |\Sigma|^{-\left(\frac{v_p+k}{2}+1\right)} \exp \left\{ -\frac{1}{2} \text{tr}(\vartheta_p \Sigma^{-1}) \right\} \quad (30)$$

The joint distribution of μ and Σ is given as Normal Inverse Wishart specified as;

$$P(\mu, \Sigma) \propto |\Sigma|^{-\left(\frac{v_p+k}{2}+1\right)} \exp \left\{ -\frac{1}{2} \text{tr}(\vartheta_p \Sigma^{-1}) \right\} -\frac{k_p}{2} (\mu - \mu_p)^T \Sigma^{-1} (\mu - \mu_p) \quad (31)$$

The initial estimates for the mean μ_p were derived from historical data, aiming to incorporate all the information from the past. The likelihood series $\bar{y}$ was based on the refined data to ensure robust resistance from outliers. The updated expected value of the covariance matrix is a weighted shrinkage combination of prior and the new sample data. The Posterior Distributions of the parameters provide the whole distribution of the optimal portfolio weights.

G. Optimal Weight

Optimal weights are the proportions of each asset that, subject to certain criteria, minimize risk or maximize return. The goal of a minimum variance portfolio is to find the asset weights that lower the portfolio's volatility by lowering the variation of the whole portfolio. This process involves creating the covariance matrix Σ , which shows how the returns of the assets move together, and the weight vector w , which shows how much of each asset is in the portfolio. The optimal weight is estimated by first obtaining the covariance matrix (Σ) from historical returns of the assets followed by finding the objective function $w^T \Sigma w$, where w^T signifies the transpose

of the weight vector w . The optimization is constrained to the criterion that the sum of the weight is equal to one.

H. Risk Free Rate

The yield on government bonds, like Treasury bills or 10-year bonds, is usually the risk-free rate. This is the possible return on an investment that has no chance of losing money. It is the lowest return that investors expect from any investment because it is almost certain. It is also used as a standard for judging how well an investment is doing and for making financial decisions. The average yield rate of Nigerian 10-year bonds is used to figure out the risk-free rate. 10-year bond risk-free rate is chosen because of its strong criteria that make it more reliable and useful. The yield rate changes based on market conditions, which show how investors feel and what they think the economy will do. The coupon rate, on the other hand, is fixed at issuance. Because it changes with market conditions, the yield rate is a better way to compare financial models. This means that it accurately shows the true cost of borrowing and the return on a risk-free investment.

I. Performance Evaluation Statistics

(a). Sharpe Ratio: compares the investment's return with its risk. It captures the extra returns earned per unit standard deviation and uses total volatility as the risk metric treating the upside and downside variability symmetrically. It is expressed mathematically below;

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (32)$$

where R_p denotes assets return of the portfolio

R_f denotes risk free rate

σ_p standard deviation of portfolio returns

(b). Sortino Ratio: It is a risk adjusted performance metric that measures the rate of return of an investment generated per unit downside risk. It is estimated as expressed below;

$$\text{Sortino Ratio} = \frac{R_p - \text{MAR}}{\sigma_d} \quad (33)$$

where R_p denotes assets return of the portfolio

MAR signifies the minimum acceptable return

σ_d represents the downside standard deviation of return.

The downside deviation measures the volatility of the asset returns that falls below the minimum acceptable return. The lower the downside deviation the lesser the risk.

J. Value at Risk (CVaR) using Variance Covariance Approach

The Var via variance covariance method of portfolio evaluation metrics calculates the maximum expected loss of a portfolio over a set of time and confidence level. It is expressed as follows;

$$\text{VaR} = z_\alpha \sigma_p \sqrt{t} W_0 \quad (34)$$

Where z_α denotes the critical value from the standard normal distribution corresponding to confidence level α , σ_p denotes portfolio standard deviation from the variance covariance matrix, t signifies adjusted time horizon and W_0 represent initial portfolio value.

K. Conditional Value at Risk (CVaR) using Variance Covariance Approach

CVaR computes the expected loss exceeding the VaR threshold.

CVaR is expressed mathematically as:

$$CVaR_{V-R} = Var_{\alpha} + \frac{\sigma_p}{1-\alpha} \gamma(z_{\alpha}) \quad (35)$$

Where σ_p denotes portfolio standard deviation, Z_{α} signifies the critical value of the normal distribution at confidence interval level α . $\gamma(\cdot)$ is the probability density function of the standard Normal distribution. CVaR using the variance covariance matrix assesses Bayesian portfolio performance by combining parameter uncertainty from posterior distributions with parametric tail loss estimations. It assumes normal return distributions; the portfolio variance derived from Bayesian covariance matrix scales the parametric VaR and expected tail loss.

IV. RESULTS AND DISCUSSION.

The dataset used in this study was retrieved from a comprehensive financial research and analysis platform (Capital IQ: <https://www.spglobal.com/marketintelligence>), which is provided by S&P Global Market Intelligence. The study identified the 30 constituent members with the highest market capitalization (NGX 30 index) on the Nigeria Stock Exchange as of June 13, 2024, and the daily closing return for each company from 2018 to 2024. Two of these companies, Geregu, and Bua foods were listed in 2022, which made them excluded from the sample, leaving the study with 28 large, liquid stocks in our sample. The 28 stocks include ["Accesscorp", "Airtelafri", "Bua cement", "Dangcem", "Dangsugar", "ETI", "FBNH", "FCMB", "Fidelitybk", "Flourmill", "GTCO", "Guinness", "Intbrew", "Mtn", "Nb", "Nestle", "Okomu oil", "Presco", "Seplat", "Stanbic", "Sterling Ng", "Total", "Transcorp", "Transcohot", "UBA", "Ucap", "WAPCO", "Zenith bank"].

Table 1: Sample of the First 9 Stocks and Closing Prices for the top 30 Stocks from 2018 to 2024

	Date	BUAC EMENT	ACCES S CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDELI TY	...
0	6/19/2018 0:00	38.95	10.35	67.8	239.0	19.4	20.00	10.75	2.19	2.16	...
1	6/20/2018 0:00	38.95	10.30	67.8	239.0	19.4	20.20	10.70	2.19	2.18	...
2	6/21/2018 0:00	38.95	10.40	67.8	230.0	19.0	20.20	10.60	2.28	2.28	...
3	6/22/2018 0:00	38.95	10.40	67.8	225.0	19.0	20.20	10.65	2.19	2.28	...
4	6/25/2018 0:00	38.95	10.15	67.8	230.0	19.0	20.00	10.70	2.11	2.22	...
...	...	...	...	...	...	...	...	...	...	...	...
1521	6/5/2024 0:00	143.20	17.15	122.0	650.7	47.0	22.00	22.50	7.90	10.80	...

Table 1 shows the daily closing prices of the first nine stocks from 2018 to 2024 for the first nine stocks.

B. Risk-Free Rate

This study, averages the 10-year bond yield over multiple years from 2018 to 2024 as the risk-free rate for the analysis, this approach mitigates the impact of short-term volatility and provides a stabilized rate that better represents the economic environment over the period of study.

Table 2: Nigeria 10-year bond yield rate for 2018 to 2024 and the risk-free rate

Year	2024	2023	2022	2021	2020	2019	2018	Average RFT
Nigeria 10- year bond yield	18.56	14.36	12.54	11.84	8.94	13.85	14.34	13.49

Table 2 above shows the 10-year bond yield rate in the period of 2018-2024, the average of this yield, 13.49% is the risk-free rate used in this study.

C. Mean-Variance Method

Using the mean and variance of each company's stock returns, the covariance matrix for the 28 companies was computed. The goal was to make the portfolio as minimum as possible employing the mean-variance approach, to create the best portfolio using the daily return data from the last six years.

Table 3: Daily Percentage Changes for 9 of the Sampled Stocks Over 1525 Days

	ACCESS CORP	AIRTE LLAFRI	BUA CEMENT	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDE LITY	FLOUR MILL	...
1	-0.005	0.000	0.000	0.000	0.000	0.018	-0.005	0.000	0.009	0.006	
2	0.010	0.000	0.000	-0.023	0.000	0.000	-0.009	0.041	0.046	0.002	
3	0.000	0.000	0.000	0.000	0.000	0.000	0.005	-0.039	0.000	0.000	
4	-0.024	0.000	0.000	0.000	0.000	0.000	0.005	-0.037	-0.026	-0.011	
5	0.020	0.000	0.000	0.000	0.000	0.000	0.014	0.090	0.050	0.000	
...	...	...	...	...	...	...	...	...	...	...	
1475	0.000	0.000	0.000	0.000	0.000	0.000	0.007	-0.006	0.064	0.000	
1476	0.003	0.000	0.000	0.000	0.000	0.000	0.011	0.000	-0.097	0.000	
1477	0.003	0.000	0.000	0.000	0.000	0.120	0.000	0.000	-0.056	0.000	
1478	0.099	0.000	0.000	0.000	0.000	0.019	0.000	-0.006	0.065	0.127	

The daily returns for each stock are shown in Table 3. These returns can be positive, indicating the stock price increased over the previous closing date, negative, indicating the stock price decreased over the previous closing date, or neutral, indicating the stock closing price remained unchanged. The efficient frontier graph that illustrates the connection between volatility (risk) and expected return is shown in Figure 1. Each point on the graph, which plots expected reward on the y-axis and volatility (measured by standard deviation of returns) on the x-axis, represents a distinct combination of stocks from the NGX 30.

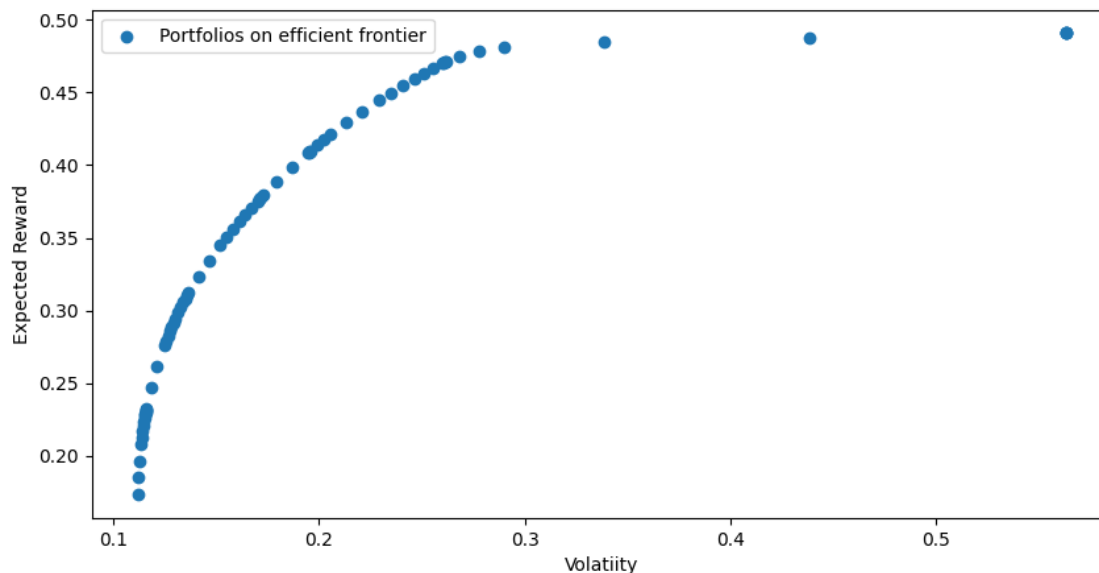


Figure 1: Efficient Frontier Graph of Expected Reward vs Volatility

The set of portfolios that offer the highest projected return for a particular level of risk or the lowest risk for a specific level of return is represented by the graph's curve. While portfolios in the lower-right portion of the curve offer less volatility at the expense of possible profits, those in the upper-left portion of the curve usually offer larger projected returns in relation to their risk. Covariance matrix for 10 out of 28 stocks are presented in Table 4.

Table 4: Covariance Matrix for 10 Out of 28 Stocks

	BUA CEMENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDE LITY	FLOUR MILL
BUA CEMENT	0.091	-0.001	-0.005	0.015	0.002	0.003	-0.003	-0.001	0.002	0.002
ACCESS CORP	-0.001	0.190	0.023	0.006	0.047	0.064	0.092	0.078	0.083	0.003
AIRTEL AFRICA	-0.005	0.023	0.201	-0.005	0.006	0.007	0.012	0.013	0.011	0.007
DANG CEM	0.015	0.006	-0.005	0.108	0.009	0.006	0.011	0.001	0.000	0.012
DANG SUGAR	0.002	0.047	0.006	0.009	0.207	0.005	0.043	0.052	0.042	0.034
ETI	0.003	0.064	0.007	0.006	0.046	0.236	0.062	0.050	0.057	0.022
FBNH	-0.003	0.092	0.012	0.011	0.043	0.062	0.238	0.082	0.084	0.031
FCMB	-0.001	0.078	0.013	0.001	0.052	0.050	0.082	0.272	0.088	0.028
FIDE LITY	0.002	0.083	0.011	0.000	0.042	0.057	0.084	0.088	0.231	0.033
FLOUR MILL	0.002	0.025	0.007	0.012	0.034	0.022	0.031	0.028	0.033	0.163

Table 5 presents the optimal allocation of stocks in our portfolio based on the minimum volatility strategy derived from mean-variance optimization. The minimum volatility method, which seeks to build a portfolio with the least amount of risk while obtaining a reasonable expected return, is used to determine these weights.

Table 5: Weights Assigned to each Stock for Constructing the Optimal Portfolio based on Mean-Variance Optimization

<u>TOTAL</u> 13.39%	<u>BUACEMENT</u> 11.13%	<u>NESTLE</u> 10.31%	<u>OKOMUOIL</u> 8.41%	<u>TRANSCOHOT</u> 8.26%	<u>MTN</u> 8.16%	<u>DANGCEM</u> 6.03%
<u>SEPLAT</u> 4.99%	<u>PRESCO</u> 4.73%	<u>GUINNESS</u> 4.72%	<u>AIRTELAFRI</u> 4.60%	<u>STANBIC</u> 4.32%	<u>FLOURMILL</u> 2.62%	<u>NB</u> 2.23%
<u>GTCO</u> 2.03%	<u>UCAP</u> 1.60%	<u>INTBREW</u> 1.38%	<u>DANGSUGAR</u> 1.27%	<u>WAPCO</u> 1.14%	<u>FCMB</u> 0.47%	<u>ETI</u> 0.17%
<u>FIDELITYBK</u> 0.06%	<u>ACCESSCORP</u> 0.00%	<u>FBNH</u> 0.00%	<u>STERLINGNG</u> 0.00%	<u>TRANSCORP</u> 0.00%	<u>UBA</u> 0.00%	<u>ZENITHBANK</u> 0.00%

Each weight indicates the percentage of the entire portfolio that is allocated to that specific stock. As an illustration, Total has the largest weight (13.39%), which suggests that it will likely have a lower risk contribution than the other stocks in our pick.

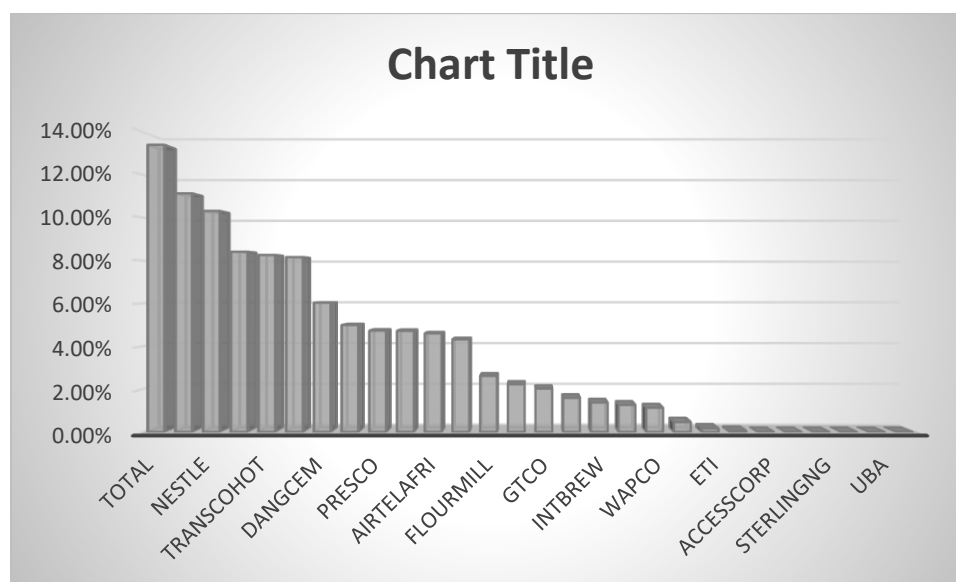


Figure 2: Graphical Representation of the Weight Assigned to each Stock to Build an Optimal Portfolio using Mean-variance Optimization

The weighting of the several stocks that make up a perfect portfolio utilizing minimum variance is more clearly shown in the pie chart above. Standard deviation is used to evaluate the risk level connected with the minimum volatility strategy. A more consistent investment profile is suggested by a lower standard deviation, which shows less return variation. As can be seen in Table 6, the minimum volatility approach resulted in a risk level of 11.24% as measured by the standard deviation of returns. This means that the annual volatility of the portfolio returns is not very high, which is consistent with the goal of the strategy to achieve stable returns and avoid risk. The least volatility method gave a projected return of 17.32%. This was done by averaging the past returns of each stock in the portfolio and annualizing the average to give an annual projection.

The portfolio is expected to return 17.32% over a year based on historical performance. This is estimated by obtaining the weighted average of the historical returns of the portfolio's constituent assets, where the weights are equivalent to the proportion of the total investment given to each stock.

Table 6: Mean-Variance Optimization Metrics. (Portfolio Statistics)

Portfolio variance	Portfolio Std. Dev	Expected Risk	Risk Rate Free	Expected Reward	Sharpe Ratio
0.01262998	0.11238317	11.24	13.49	17.32	0.34

The Sharpe ratio of 0.34 means that the portfolio is generating 0.34 units of return above the risk-free rate for every unit of risk taken. This trade-off between risk and return is the essence of good portfolio optimization where one tries to maximize return for a given level of risk or minimize risk for a given level of return.

C. Bayesian Optimization Method

The portfolio's performance was further enhanced by leveraging probabilistic models and iterative evaluation via Bayesian optimization with the aim of identifying the optimal allocation of assets that balances risk and return effectively. Table7 reveals the sample closing price of the top 30 stocks from 2018 to 2024 for the first 10 stocks.

Table 7: Sample closing price of the top 30 stocks from 2018 to 2024 for the first 10 stocks

	BUA CEMENT	ACCESS CORP	AIRTEL LAFRI	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDELITY	FLOUR MILL	...
0	38.95	10.35	67.8	239.0	19.4	20.00	10.75	2.19	2.16	31.15	
1	38.95	10.30	67.8	239.0	19.4	20.20	10.70	2.19	2.18	31.35	
2	38.95	10.40	67.8	230.0	19.0	20.20	10.60	2.19	2.28	31.40	
3	38.95	10.40	67.8	225.0	19.0	20.20	10.65	2.28	2.28	31.00	
4	38.95	10.15	67.8	230.0	19.0	20.00	10.70	2.11	2.22	31.05	
...											
1521	143.20	17.15	122.0	656.7	47.0	22.0	22.50	7.90	10.80	38.00	
1522	143.20	17.20	122.3	656.7	47.0	22.75	22.75	7.90	9.75	38.00	
1523	143.20	17.25	120.2	656.7	47.0	23.85	23.00	7.90	9.20	38.00	
1524	143.20	18.95	119.3	656.7	47.0	24.30	23.00	7.85	9.80	41.80	
1525	143.20	19.30	117.7	656.7	47.0	23.70	22.90	7.85	10.00	41.80	

1526 rows × 28 columns.

Bayesian updating allows the combination of prior and likelihood data to obtain the posterior distribution, representing the revised beliefs about the expected returns and covariances after considering the new information. The posterior mean vector and the posterior covariance matrix are derived using weightings that incorporate both prior and likelihood statistics.

Table 8: Prior Data

Date\ PRIOR DATA	BUAC EMENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDEL ITY	FLOUR MILL	...
2018-06-19	38.95	10.35	67.8	239.0	19.4	20.0	10.79	2.19	2.16	31.15	...
2018-06-20	38.95	10.30	67.8	239.0	19.4	20.2	10.70	2.19	2.18	31.35	...
2018-06-21	38.95	10.40	67.8	230.3	19.0	20.2	10.60	2.28	2.28	31.40	...
2018-06-22	38.95	10.40	67.8	225.0	19.0	20.2	10.65	2.19	2.28	31.00	
2018-06-25	38.95	10.15	67.8	230.0	19.0	20.0	10.70	2.11	22	31.05	...
...	...	...	...	...	...	...	...	...	...	...	...
Date\ PRIOR DATA	SEPL	STANBIC	STERLI NGING	TOTAL	TRANS CORP	TRANS COHOT	UBA	UCAP	WAPCO	ZENITH	
2018-06-19	140.25	41.785	1.37	193.3	1.45	7.45	10.60	3.07	39.0	122.7	
2018-06-20	141.75	42.000	1.36	193.3	1.49	7.45	10.75	3.11	39.0	122.0	
2018-06-21	141.50	41.785	1.38	193.3	1.50	7.45	10.70	3.15	40.0	121.9	
2018-06-22	140.75	41.785	1.36	193.3	1.46	7.45	10.60	3.22	39.0	122.0	
2018-06-25	139.25	42.000	1.40	193.3	1.44	7.45	10.60	3.26	38.1	122.5	

The Table 8 and Table 9 depicts the prior data which consists of historical returns from 2018 to 2023 and likelihood data from 2023 to 2024 respectively. This period captures a broad range of market conditions, providing a solid foundation for initial beliefs about the expected returns and covariances of the assets in the NSE 30 index.

Table 9: Likelihood Data

Date\ LIKELIHOOD DATA	BUAC EMENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCM B	FIDE LITY	FLOU RMIL L	...
2018-06-19	92.0	14.30	132.4	286.8	23.0	13.85	14.50	5.0	6.09	34.75	...
2018-06-20	86.0	15.70	133.7	284.0	25.3	15.20	15.95	5.4	6.69	35.00	...
2018-06-21	86.0	14.25	129.4	284.0	24.0	15.25	16.10	5.0	6.91	35.00	...
2018-06-22	86.0	14.25	124.8	284.0	23.0	15.40	15.80	5.0	6.94	35.00	
2018-06-25	86.0	14.80	123.5	286.9	23.5	15.60	15.90	5.06	7.25	34.95	...
...	...	...	...	...	...	...	...	...	...	...	...
Date\ LIKELIHOOD DATA	SEPL	STANBI C	STERLIN GING	TOTAL	TRANS CORP	TRANS CO HOT	UBA	UCAP	WAP CO	ZENI TH	
2018-06-19	119.0	49.20	2.50	278.3	3.22	11.00	10.90	14.20	28.10	17.35	
2018-06-20	119.4	54.10	2.75	306.1	3.53	12.10	11.95	15.30	29.80	17.35	
2018-06-21	120.8	54.00	3.02	336.7	3.37	13.31	11.10	14.30	28.75	16.69	
2018-06-22	118.2	52.0	2.95	336.7	3.50	14.60	11.10	13.95	28.20	17.02	
2018-06-25	121.6	52.95	2.95	336.7	3.30	14.60	11.50	14.30	28.70	17.86	

The prior statistics include the average returns and the covariance matrix over this period; these statistics represent initial assumptions before considering any new data. The likelihood period reflects the latest market conditions, which may differ significantly from historical trends. The likelihood statistics include the average returns and the covariance matrix over this period. The likelihood data serves as new evidence to update the prior beliefs.

Table 10: Covariance Matrix for 10 out of 28 Stocks for Prior Data

	BUACE MENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDE LITY	FLOUR MILL	...
BUACEMENT	0.070	0.000	-0.003	0.006	0.003	0.004	-0.001	0.003	0.002	0.001	...
ACCESSCORP	0.000	0.162	0.023	0.006	0.027	0.055	0.064	0.052	0.066	0.015	...
AIRTEL AFRICA	-0.003	0.023	0.227	-0.005	0.005	0.009	0.014	0.014	0.011	0.004	...
DANGCEM	0.006	0.006	-0.005	0.097	0.007	0.006	0.008	-0.001	0.000	0.012	...
DANGSUGAR	0.003	0.027	0.005	0.007	0.147	0.044	0.035	0.028	0.032	0.019	...
ETI	0.004	0.055	0.009	0.006	0.044	0.231	0.054	0.040	0.052	0.022	...
FBNH	-0.001	0.064	0.014	0.008	0.035	0.054	0.171	0.054	0.067	0.022	...
FCMB	0.003	0.052	0.014	-0.001	0.028	0.040	0.054	0.247	0.069	0.019	...
FIDELITY	0.002	0.066	0.011	0.000	0.032	0.052	0.067	0.069	0.195	0.032	...
FLOURMILL	0.001	0.015	0.004	0.012	0.019	0.022	0.022	0.019	0.032	0.149	...
GTCO	0.002	0.047	0.007	0.007	0.030	0.047	0.044	0.040	0.045	0.011	...

Table 10 shows the covariance matrix of the prior data (2018 to 2023) gotten from the daily returns, whereas Table 11 shows the percentage change of the likelihood data which would be needed to calculate the covariance. It denotes the daily returns on each asset with likelihood data.

Table 11: Daily Percentage Changes for 10 of the Sampled Stocks Over 253 Days (2023 to 2024)

Date	BUAC EMENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDEL ITY	FLOUR MILL	...
14/06/2023	-0.065	0.098	0.010	-0.010	0.100	0.097	0.100	0.080	0.099	0.007	...
15/06/2023	0.000	-0.092	-0.032	0.000	-0.051	0.003	0.009	-0.074	0.033	0.000	...
16/06/2023	0.000	0.000	-0.036	0.000	-0.042	0.010	-0.019	0.000	0.004	0.000	...
19/06/2023	0.000	0.039	-0.010	0.010	0.022	0.013	0.006	0.000	0.045	-0.001	...
20/06/2023	0.000	0.003	-0.015	0.001	0.021	-0.013	0.003	0.020	-0.034	0.000	...
...	...	...	...	...	...	...	...	...	...	...	...

Table 12: Covariance Matrix for 10 out of 28 Stocks for Likelihood Data

	BUACE MENT	ACCESS CORP	AIRTEL AFRICA	DANG CEM	DANG SUGAR	ETI	FBNH	FCMB	FIDELITY	FLOURMILL	...
BUACE MENT	0.193	-0.006	-0.014	0.059	-0.002	0.005	-0.012	-0.024	-0.002	0.003	...
ACCESS CORP	-0.006	0.331	0.021	0.004	0.148	0.109	0.234	0.204	0.163	0.082	...
AIRTEL AFRICA	-0.014	0.021	0.074	-0.006	0.012	-0.004	0.001	0.004	0.013	0.021	...
DANG CEM	0.059	0.004	-0.006	0.160	0.013	0.003	0.023	0.012	-0.003	0.014	...
DANG SUGAR	-0.002	0.148	0.001	0.013	0.506	0.052	0.082	0.171	0.088	0.108	...
ETI	-0.005	0.109	-0.004	0.003	0.052	0.260	0.098	0.102	0.079	0.023	...
FBNH	0.012	0.023	0.001	0.023	0.082	0.098	0.102	0.217	0.166	0.075	...
FCMB	-0.024	0.204	0.000	0.012	0.171	0.102	0.217	0.394	0.183	0.073	...
FIDE LITY	-0.002	0.163	0.013	-0.003	0.088	0.079	0.166	0.183	0.409	0.040	...
FLOUR MILL	0.003	0.082	0.021	0.014	0.108	0.022	0.075	0.073	0.040	0.233	...
...	...	...	...	...	...	...	...	...	...	...	...

Table 12 illustrates the covariance matrix derived from the prior data spanning 2023 to 2024, calculated from daily stock returns. This matrix provides a comprehensive view of the interrelationships between the different assets in the dataset, highlighting how the returns of each pair of assets move together. The covariance matrix gives critical insights into the correlations and dependencies among the assets; the optimal weight was derived for the prior and likelihood data using minimum volatility by identifying the allocation that minimizes the overall portfolio risk while maintaining the desired level of returns in the face of market fluctuations. Three different portfolios were created for this study with Bayesian inference; each giving varying weightings to the prior and likelihood data to derive the posterior optimal weights and returns. The arbitrary weights were chosen to integrate the different parameter uncertainty through various posterior predictive distribution. This method accounts for estimation error, reducing portfolio weight instability by estimating posterior with different ratio of prior with observed market data to quantify uncertainty. The objective is to evaluate the robustness of the optimization process by comparing different Bayesian optimization models with various weighting portfolio. The portfolios reflect different levels of emphasis on historical data versus recent trends. This approach enables exploration of varying degrees of reliance on past versus present information affect portfolio optimization, leading to more resilient and adaptive investment strategies.

D. Portfolio 1

Portfolio 1 was constructed adopting an equal weighting strategy to both prior and likelihood periods in the optimization process. The portfolio leverages historical data from two distinct periods: the prior period (2018-2023) and the likelihood period (2023-2024), each weighted at 50% by balancing the influence of historical performance with recent market dynamics in portfolio

construction. Portfolio 1 seeks to enhance portfolio stability and resilience against market volatility ensuring a robust investment strategy aligned with financial objectives.

Table 13: Weights Assigned to each Stock for Constructing the Optimal Portfolio based on Bayesian Optimization Portfolio 1

Total 12.14%	Nestle 11.84%	AirtelAfrica 9.23%	Buacement 8.71%	Okomuoil 8.27%	Transcorphot 7.32%	Seplat 6.06%
Dangcem 5.72%	MTNN 5.48%	Presco 5.06%	Guinness 4.01%	NB 2.73%	Stanbic 2.60%	Interbrew 1.88%
Ucap 1.75%	Flourmill 1.47%	Dangsugar 1.22%	Wapco 1.08%	GTCO 1.07%	ETI 0.74%	Transcorp 0.41%
FCMB 0.41%	Fidelitybk 0.40%	Sterlingng 0.38%	Accesscorp 0.00%	FBNH 0.00%	UBA 0.00%	Zenithbank 0.00%

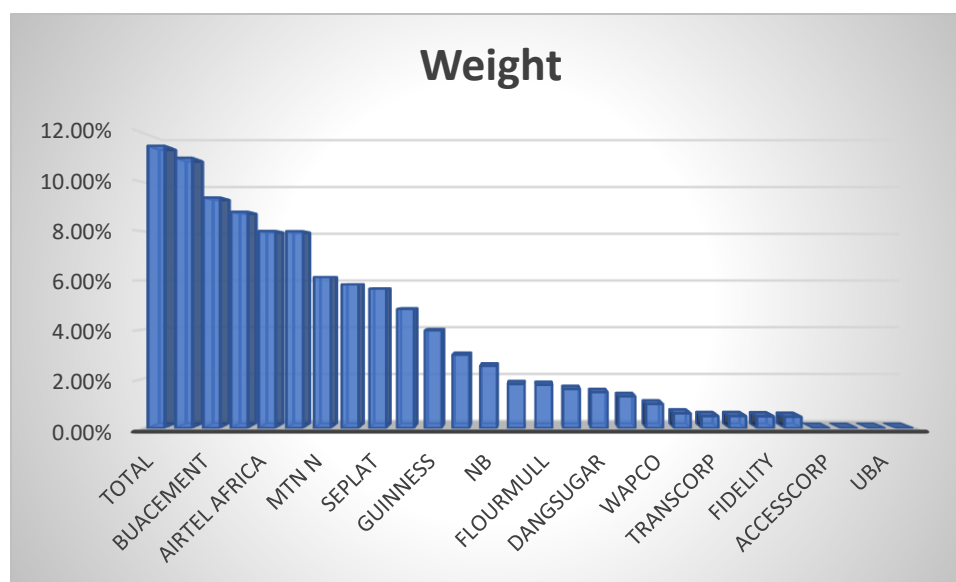


Figure 3. Graphical Representation of the Weight Assigned to each Stock to Build an Optimal Portfolio using equally Weighted Prior and Likelihood Data

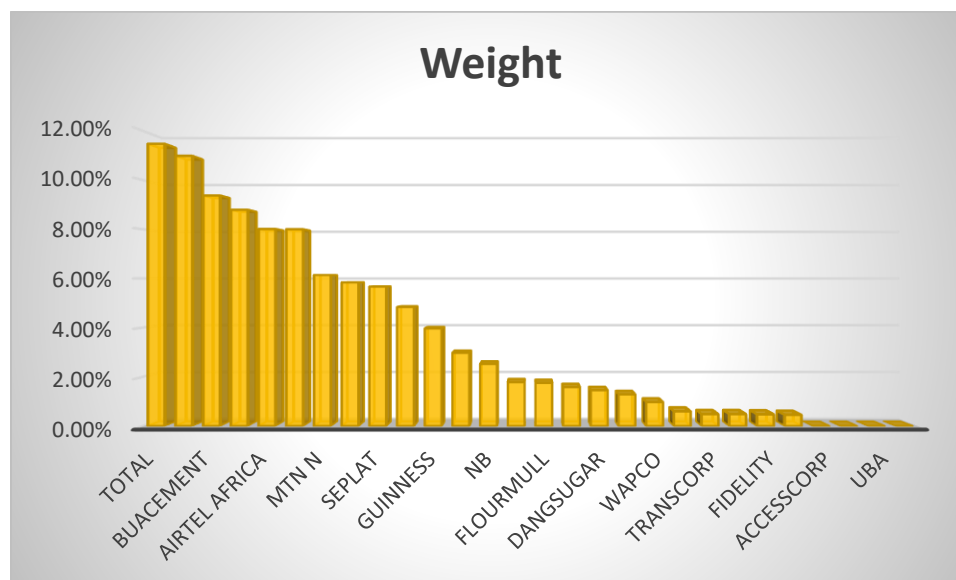
Based on past performance and current market expectations, the expected return of Portfolio 1 is 16.96%. The standard deviation of the portfolio, i.e. the risk is 11.83%. This measure reflects average volatility around the expected return with a combination of risk and return objectives. Portfolio 1 has a Sharpe ratio of 0.29, indicating a good risk-adjusted return. This ratio is a good performer against volatility as it is able to provide returns in relation to the taken risk.

E. Portfolio 2

Portfolio 2 is not equally weighted, 60% was allocated to the prior period (2018-2023) to establish baseline performance metrics and covariance structure and 40% to the likelihood period (2023-2024) to reflect updated market conditions and performance trends. Portfolio 2 aims to use more historic performance data and recent market insights to develop diversified asset allocations. It constructs a portfolio that is balanced for optimizing risk-adjusted returns, optimizing risk and return profiles and managing potential risks.

Table 14: Weights assigned to each stock for constructing the optimal portfolio based on Bayesian optimization portfolio 2

Total 11.47%	Nestle 10.98%	Buacement 9.37%	Transcohot 8.79%	Airtelafrika 8.01%	Okomuoil 8.00%	MTNN 6.14%
Dangcem 5.84%	Seplat 5.67%	Presco 4.84%	Guinness 3.97%	Stanbic 2.98%	NB 2.53%	Intbrew 1.79%
Flourmill 1.76%	UCAP 1.59%	Dangsugar 1.46%	GTCO 1.29%	Wapco 0.98%	ETI 0.60%	Transcorp 0.49%
FCMB 0.49%	Fidelitybk 0.48%	Sterlingng 0.46%	Accesscorp 0.00%	FBNH 0.00%	UBA 0.00%	Zenithbank 0.00%

**Figure 4.** Graphical Representation of the Weight Assigned to each Stock to Build an Optimal Portfolio using 60% Weighted prior and 40% likelihood Data

The second portfolio is constructed on the basis of a large volume of historical data and the most recent data from the market with an expected return of 17.68%, which demonstrates a strong expected average rate of return on the basis of the combined performance statistics of the two periods. This expected return is an estimate of the portfolio's potential for profit, considering the long-term trends and recent market developments. The standard deviation, used as a measure of risk level of the portfolio, is 11.84%. The volatility of the portfolio returns is the standard deviation which shows how much the actual returns are likely to deviate from the expected return. The standard deviation of 11.84% indicates a moderate level of risk, striking a balance between the need for returns and the potential for performance fluctuations. Additionally, Portfolio 2 obtains a Sharpe ratio of 0.35, the Sharpe ratio quantifies the risk-adjusted return of the portfolio, indicating the amount to which investors are compensated for the risk they bear. A Sharpe ratio of 0.35 indicates that Portfolio 2 has a good return to risk ratio and it is a good risk adjusted performance.

F. Portfolio 3

Portfolio 3 uses a weighted optimization approach, factoring in current trends and conditions that could influence future results, with 40% allocated to the previous period (2018-2023) and 60% to the probability period (2023-2024). The approach focuses on the latest market trends, but also considers historical performance data for diversified asset allocations. The portfolio optimization process brings together data from these two different periods, with the historical data used to establish baseline performance and covariance structure and the recent data to reflect the current market environment and trends in performance. Portfolio 3 aims to construct a dynamic portfolio that maximizes risk-adjusted return by considering more recent information from the market.

This method enables the portfolio to be more responsive to recent changes and opportunities while still benefiting from historical insights.

Table 15: Weights Assigned to each Stock for Constructing the Optimal Portfolio based on Bayesian Optimization Portfolio 3

Total 12.80%	Nestle 12.69%	Airtelafrica 10.45%	Okomuoil 8.54%	Buacement 8.05%	Seplat 6.45%	Transcohot 5.86%
Dangcem 5.62%	Presco 5.28%	MTNN 4.80%	Guinness 4.05%	NB 2.93%	Stanbic 2.23%	Intbrew 1.97%
Ucap 1.90%	Wapco 1.18%	Flourmill 1.18%	Dangsugar 0.98%	ETI 0.90%	GTCO 0.86%	Transcorp 0.34%
Fcmb 0.33%	Fidelitybk 0.32%	Sterlingng 0.31%	Accesscorp 0.00%	FBNH 0.00%	UBA 0.00%	Zenithbank 0.00%

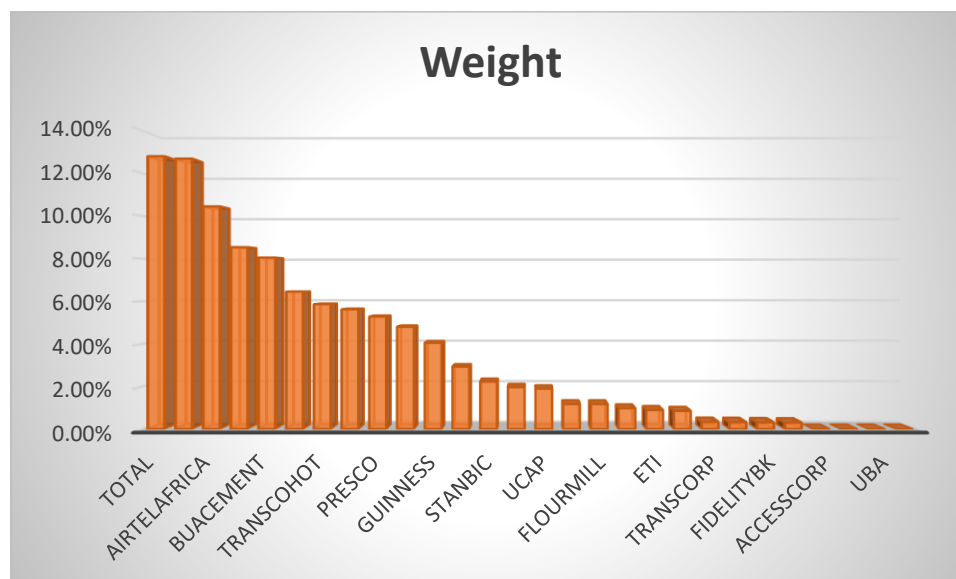


Figure 5. Graphical Representation of the Weight Assigned to each Stock to Build an Optimal Portfolio using 40% Weighted Prior and 60% likelihood Data

The expected return of the portfolio based on the joint information of the historical and recent data is 16.23%. The standard deviation for portfolio 3 is 11.69% which signifies the risk. This standard deviation measures the volatility of the portfolio's returns. 11.69 %, representing a moderate level of risk. showing how well the portfolio rewards investors for taking on risk. On the other hand, the

Sharpe ratio of portfolio 3 is 0.23. The Sharpe ratio measures the risk-adjusted return of the portfolio, showing the extent to which the portfolio rewards investors for the risk they undertake.

Bayesian Portfolio optimization generally improves upon mean variance optimization by treating the risk inputs and unknown returns as random variables rather than being fixed. The Bayesian framework used in this research incorporates parameter uncertainty through various weights of priors on expected returns and portfolio concentration constrain. Table 16 reveals the performance of mean variance optimization portfolio and various Bayesian optimization portfolios. Notably, Bayesian Portfolio 2 exhibit a higher Sharpe ratio and Sortino ratio with 0.35 and 1.04 respectively indicating attractive returns adjusted for risk performance. Also, both VaR and CvaR of the portfolios increases as the confidence level rises from 95% to 99%, reflecting a more conservative assessment of portfolio risk.

Table 16: Performance Evaluation Metrics

	MVO	Bayesian-Portfolio 1	Bayesian-Portfolio 2	Bayesian-Portfolio 3
Sharpe Ratio	0.34	0.29	0.35	0.23
Sortino Ratio	0.59	0.62	1.04	0.49
VaR 95%	11.59	12.04	11.85	11.92
VaR 99%	16.24	16.87	16.18	16.60
CVaR 95%	14.90	15.12	14.74	14.27
CVaR 99%	19.68	19.39	19.83	19.22

Results of various portfolio optimization weights above were assessed via out-of-sample performance in Table 17, the out of sample validation utilizes the timeframe of July 16, 2024 to July 16, 2025, Bayesian models reported lower variance and higher Sharpe Ratio. Bayesian Portfolio 2 is more efficient as it will extract higher excess return per unit standard deviation. This stage serves to test the models' practicality and their ability to generalize beyond the data they were trained on.

Table 17: Out of Sample Validation Metrics

	MVO	Bayesian-Portfolio 1	Bayesian-Portfolio 2	Bayesian-Portfolio 3
Expected Return	16.79	16.84	16.93	16.86
Variance	12.05	11.56	11.40	11.78
P value	0.289	0.021	0.003	0.057
Sharpe Ratio	0.416	0.440	0.451	0.430

V. CONCLUSION

In this study, mean-variance portfolio optimization approach and Bayesian portfolio optimization approach are used to construct minimum variance portfolios for NSE 30 stocks. The project involves the construction of optimization models, evaluation of statistical studies on historical market data, and the review of recent data, effects and market expectations. The results show the flexibility of Bayesian portfolios for creating portfolios that may yield a higher output of returns through different weights of historical and recent data. In general, the choice of such portfolios will depend on the risk tolerance and expected returns of the investor. Due to its better risk-adjusted performance and higher returns, portfolio 2 is the best choice for investors who want maximum returns with acceptable risk. The mean-variance portfolio will be beneficial to those that prefer low risk and still maintain competitive returns. Portfolio 3 is most suitable for someone who wants stability and less volatility. Portfolio 1 is a more balanced option. Mean variance portfolio Optimization is estimated by obtaining the weighted average of the historical returns of the portfolio's constituent assets while Bayesian Portfolio optimization update returns expectations dynamically as new information arrives. Overall, evidence of the research suggests that Bayesian Portfolio optimization gives a flexible framework for addressing parameter uncertainty, volatility patterns and definite risk concentration. The findings of the research support the work of Xinyu (2024) that concluded in his research on portfolio optimization that Bayesian approach exhibited a superior performance, showcasing robust resilience to potential risks.

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