

Mathematical Modelling and Sensitivity Analysis of Drug Abuse Dynamics with Awareness, Guidance & Counselling Interventions

¹Manko, A.A., ²Ibrahim, M.O., ³Kazeem I., ⁴Dotia A.K

^{1,3}Department of Mathematics and Statistics, Kwara State University, Malete, Kwara State, Nigeria

²Department of Mathematics, Faculty of Physical Science, University of Ilorin, Ilorin, Kwara State,

⁴ Department of Mathematics, Faculty of Natural and Applied Sciences, Nigerian Army University Biu, Biu, Borno State, Nigeria

Corresponding Author: abdulra2005@gmail.com

ABSTRACT

Drug abuse remains a major global public health and socioeconomic challenge, particularly among young people. This study develops a deterministic compartmental model to investigate the dynamics of drug addiction in a population under the influence of awareness programs, guidance and counselling. Qualitative analysis was conducted to establish the positivity of solutions, derive the drug-free equilibrium, and evaluate the basic reproduction number R_0 , as a threshold parameter for the persistence or elimination of drug abuse. The endemic equilibrium state was also calculated to examine long term persistence. A local sensitivity analysis revealed that parameters with negative sensitivity indices most notably the transition of active drug abusers into guidance and counseling programs act as the primary protective factor against the epidemic by reducing transmission potential as their value increase. Conversely, parameters with positive sensitivity indices, particularly those associated with initiation and human recruitment, enhance transmission potential as they increase. Finally, numerical simulations were performed to validate the analytical results and evaluate the impact of various intervention strategies.

KEYWORDS: Drug Addiction, Sensitivity Analysis, Guidance and Counselling, Awareness Program

I. INTRODUCTION

Drug abuse refers to the unlawful, inappropriate, or excessive use of substances, often defined by societal norms as problematic [7], [13]. Also, Unlawful drug administration is said to be nonmedical use of varieties of drugs that are prohibited by local/international law [4]. While drugs are primarily intended to treat health conditions [1], abusing them threatens individual health and socioeconomic progress worldwide [23]. In 2012, drug abuse resulted in around 183,000 deaths worldwide [19]. According to the [20], drug administration methods vary by substance type, with oral intake and smoking serving as the most common methods, alongside injection depending on the drugs used. The degree of drug absorption into the circulation and the associated effects depend on the mode of administration [11]. Numerous elements interact to cause drug abuse and addiction, including biological aspects like genetic predispositions, psychological factors like choice and craving processes, social factors like peer pressure, and societal ones like substance availability. Alcohol, nicotine, barbiturates, cannabis, cocaine, methaqualone, opium alkaloids, synthetic opioids, and other substances are the most often misused substances. Unfortunately, over 8% of the nation's population suffers from a drug use disorder at some point in their lives [6]. Drug abuse and addiction have become major challenges affecting communities worldwide, leading to devastating

health, social, and economic consequences. While enforcement and treatment strategies are vital in addressing drug-related issues, prevention through education stands out as a proactive and sustainable solution [12].

Drug misuse has been the subject of several studies. For example, [22] reported that initiation, progression, cessation, treatment, recovery, and relapse associated with drug use are influenced by a combination of biological, psychological, social, and environmental factors, including peer influence, family and community environments, social networks, accessibility of substances, and availability of treatment services, [15] presented a mathematical model of drug abuse. He used a seven-compartmental model in his study; his analysis, which was conducted using a mathematical model technique, reveals that anti-drug education programs and ineffective treatment have an impact on the dynamics of drug dissemination in a population. He then conducted quantitative analyses of equilibrium points, local stability, and global stability. A mathematical model was developed by [2], With six compartments susceptible, exposed, infected, guided and counselled, rehabilitated, and recovered—they created the SEIGWR model, a mathematical representation of drug dependence as an infectious disease. The presence of equilibrium points and the model's stability were established. According to the study's quantitative analysis, local stability was present when $R_0 < 1$. They concluded from the drug misuse model's numerical analysis that counselling and guidance affected its dynamics. It has been demonstrated that greater assistance and counselling lead to fewer infections.

[9] conducted a study on drug abuse dynamics among college students. The SEIPR model, which was proposed and examined, emphasized the importance of enrolling a large number of affected students in treatment programs along with measures to reduce the transmission to curb the spread of drugs eventually. [15] also introduced a mathematical model of drug abuse. In his study, he considered a seven-compartment model; his analysis, carried out, suggests the influence of ineffective treatment and anti-drug education campaigns on the dynamics of drug spread in a population and was investigated using a mathematical model approach. Another notable contributor is [14], who formulated a coupled system of two submodels: the drug-abuse-only model and the violence-only model. The drug-abuse and violence co-menace total population was divided into nine sub-classes. An optimal control of intervention strategies for a drug abuse and violence co-menace model was carried out; drug abuse only free equilibrium, drug abuse only present equilibrium, violence only free equilibrium, and violence only present equilibrium were also carried out.

This article develops and analyses a comprehensive mathematical model of drug abuse in society. It builds on the work in [8] by incorporating two susceptible human classes and an awareness, guidance, and counselling component within the human population, thereby enhancing the model's robustness and overall control strategies for curtailing drug abuse in society. The structure of the paper is as follows: Section Two describes the model formulation, the model's positivity, the drug-free and endemic equilibrium points, and the derivation of the basic reproduction number. Sensitivity analysis with results and discussion formed the concluding part.

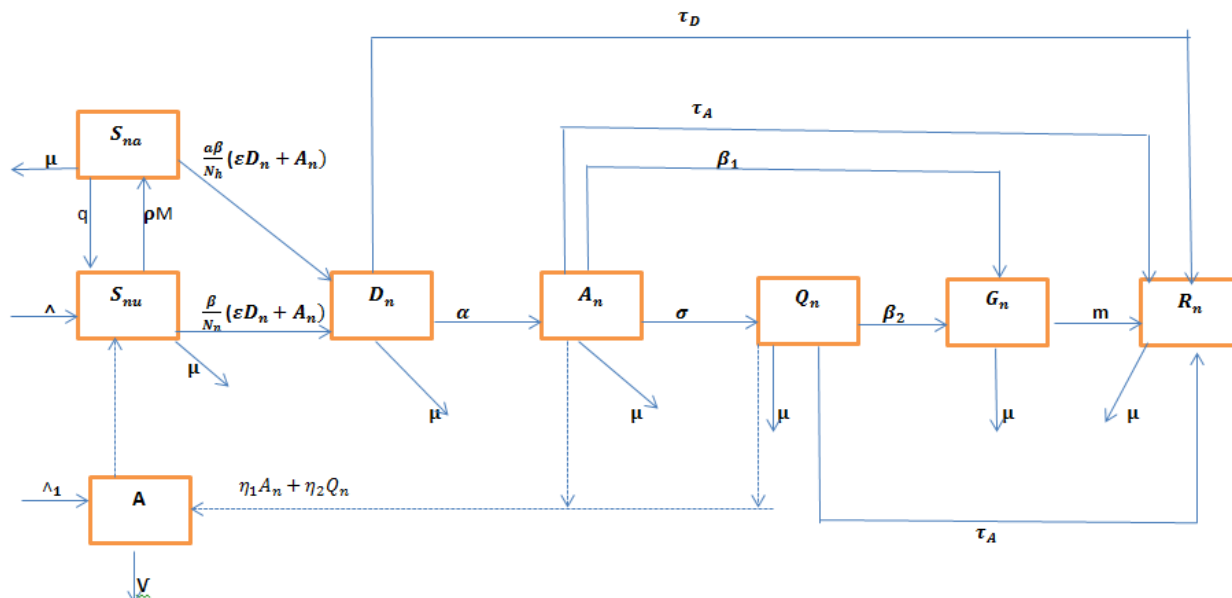
II. MODEL FORMULATION

We considered a human population affected by drug abuse and subdivided them into the following: Susceptible unaware class (S_{nu}), Susceptible aware class (S_{na}), Drug users (D_n), Drug Abusers

(A_n), Disabled/retarded addicts (Q_n), Drug users who have received guidance counselling (G_n), The recovered Class (R_n). Lastly, there is a non-human compartment (A) known as an awareness program driven by drug addiction prevalence and media coverage.

$$N(t) = S_{nu}(t) + S_{na}(t) + D_n(t) + A_n(t) + Q_n(t) + G_n(t) + R_n(t)$$

And the non-human compartment $A(t)$



From the diagram above, the model's equations are shown below.

$$\frac{dS_{nu}}{dt} = \lambda - \frac{\beta}{N_n} (\epsilon D_n + A_n) S_{nu} - \rho S_{nu} A + q S_{na} - \mu S_{nu} \quad (1)$$

$$\frac{dS_{na}}{dt} = \rho S_{nu} A - \frac{a\beta}{N_h} (\epsilon D_n + A_n) S_{na} - (q + \mu) S_{na} + \omega R_n \quad (2)$$

$$\frac{dD_n}{dt} = \frac{\beta}{N_n} (\epsilon D_n + A_n) (S_{nu} + a S_{na}) - (\alpha + \tau_D + \mu) D_n \quad (3)$$

$$\frac{dA_n}{dt} = \alpha D_n - (\tau_A + \sigma + \beta_1 + \mu) A_n \quad (4)$$

$$\frac{dQ_n}{dt} = \sigma A_n - (\tau_Q + \delta + \beta_2 + \mu) Q_n \quad (5)$$

$$\frac{dG_n}{dt} = \beta_1 A_n + \beta_2 Q_n - (m + \mu) G_n \quad (6)$$

$$\frac{dR_n}{dt} = \tau_D D_n + \tau_A A_n + \tau_Q Q_n + m G_n - (\mu + \omega) R_n \quad (7)$$

$$\frac{dA}{dt} = \lambda_1 + \eta_1 A_n + \eta_2 Q_n - v A \quad (8)$$

Table 1: Description of Variables and Parameters

Variables and Parameters	Description of Variables and Parameters
S_{nu}	Susceptible unaware individuals

S_{na}	Susceptible aware individuals
D_n	Drug users
A_n	Drug Abusers
Q_n	Disabled or retarded addicts
G_n	Drug users who have received guidance and counselling.
R_n	Recovered
A	Awareness Programmes
$\wedge$	Human recruitment
μ	Natural death
α	Progression rate from D_n to A_n
δ	Death rate due to addiction
ρ	Rate at which awareness programs move susceptible unaware to susceptible aware
τ_D	Progression rate from D_n to R_n
τ_A	Progression rate from A_n to R_n
τ_Q	Progression rate from Q_n to R_n
ρ	Rate of awareness
β	Initiation rate
ε	Relative influence of drug users compared to abusers
σ	Progression rate from A_n to Q_n
β_1	Progression rate from A_n to G_n
a	Reduction factor due to awareness
β_2	Progression rate from Q_n to G_n
$\wedge_1$	Growth influx of awareness
V	The time rate of decay of drug addiction prevalence
η_1, η_2	Awareness program stimulated rates
q	Rate at which S_{na} loses awareness
ω	Rate at which R_n loses immunity

A. MODEL ASSUMPTIONS

The deterministic Drug abuse model developed in this study is based on some of the following assumptions:

- New individuals enter the population through recruitment (birth or immigration) into the susceptible unaware class at a constant rate $\wedge$
- Drug initiation occurs through effective contact between susceptible individuals and drug users or drug abusers
- Natural death occurs in every human compartment at the rate μ
- Awareness programmes are dynamic and are represented by a non-human compartment $A(t)$
- Recovered individuals gradually lose their resistance to drug use and return to the susceptible aware class at rate ω
- All abused substances and treatment methods possess uniform characteristics and effectiveness
- Awareness programmes reduce the likelihood of drug initiation.

B. POSITIVITY OF THE MODEL

In this subsection, we establish the positivity of the system of equations (1)-(8) above.

Lemma 1. Let all parameters be non-negative and let the initial conditions satisfy

$$S_{nu}(0), S_{na}(0), D_n(0), A_n(0), Q_n(0), G_n(0), R_n(0), A(0) \geq 0.$$

Then the solution of system (1)–(8) satisfies

$$S_{nu}(t), S_{na}(t), D_n(t), A_n(t), Q_n(t), G_n(t), R_n(t), A(t) \geq 0 \text{ for all } t \geq 0.$$

Proof. From the system of equations (1) above,

$$\frac{dS_{nu}}{dt} = \Lambda - \frac{\beta}{N_n} (\varepsilon D_n + A_n) S_{nu} - \rho S_{nu} A + q S_{na} - \mu S_{nu}$$

we have:

$$\frac{dS_{nu}}{dt} \geq \Lambda - (\rho + \mu) S_{nu} \quad (9)$$

$$\frac{dS_{nu}}{dt} + (\rho + \mu) S_{nu} \geq \Lambda \quad (10)$$

Using integrating factor

$$\frac{dS_{nu}}{dt} e^{(\rho+\mu)t} + (\rho + \mu) S_{nu} e^{(\rho+\mu)t} \geq \Lambda e^{(\rho+\mu)t} \quad (11)$$

$$S_{nu} e^{(\rho+\mu)t} \geq \int \Lambda e^{(\rho+\mu)t} dt \quad (12)$$

$$S_{nu} e^{(\rho+\mu)t} \geq \frac{\Lambda}{(\rho+\mu)} e^{(\rho+\mu)t} + k \quad (13)$$

$$S_{nu}(t) \geq \frac{\Lambda}{(\rho+\mu)} + k e^{-(\rho+\mu)t} \quad (14)$$

At $t = 0$

$$S_{nu}(0) \geq \frac{\Lambda}{(\rho+\mu)} + k, \text{ that is } k = S_{nu}(0) - \frac{\Lambda}{(\rho+\mu)} \quad (15)$$

$$S_{nu}(t) \geq S_{nu}(0) e^{-(\rho+\mu)t} + \frac{\Lambda}{(\rho+\mu)} (1 - e^{-(\rho+\mu)t}) > 0 \quad (16)$$

Similarly,

$$S_{na}(t) \geq S_{na}(0) e^{-(q+\mu)t} > 0 \quad (17)$$

$$D_n(t) \geq D_n(0) e^{-(\alpha+\tau_D+\mu)t} > 0 \quad (18)$$

$$A_n(t) \geq A_n(0) e^{-(\tau_A+\sigma+\beta_1+\mu)t} > 0 \quad (19)$$

$$Q_n(t) \geq Q_n(0) e^{-(\tau_Q+\delta+\beta_2+\mu)t} > 0 \quad (20)$$

$$G_n(t) \geq G_n(0) e^{-(m+\mu)t} > 0 \quad (21)$$

$$R_n(t) \geq R_n(0) e^{-(\omega+\mu)t} > 0 \quad (22)$$

$$A(t) \geq A(0) e^{-(v)t} > 0 \quad (23)$$

Each coordinate derivative is non-negative on the corresponding boundary; therefore, all solutions of the system of (1)-(8) are positive for all $t > 0$

C. DRUG-FREE EQUILIBRIUM

Let the drug-free equilibrium of the model be

$$E^* = (S_{nu}, S_{na}, D_n, A_n, Q_n, G_n, R_n, A) = (S_{nu}^*, S_{na}^*, D_n^*, A_n^*, Q_n^*, R_n^*, A^*) \quad (24)$$

The Drug Free Equilibrium (DFE) of the dynamical system in (1-8) only exists in the absence of drugs. We can then compute DFE by setting the system of equations in (1-8) to zero and establish DFE points, where

$$D_n^* = A_n^* = G_n^* = Q_n^* = R_n^* = 0 \quad (25)$$

Hence, solving the equations (1)–(8), we have;

$$A^* = \frac{\Lambda_1}{v} \quad (26)$$

$$S_{nu}^* = \frac{\Lambda v(q+\mu)}{\mu(\rho\Lambda_1 + v(q+\mu))} \quad (27)$$

$$S_{na}^* = \frac{\rho\Lambda_1\Lambda}{\mu(\rho\Lambda_1 + v(q+\mu))} \quad (28)$$

D. ENDEMIC EQUILIBRIUM

Let

$$E^{**} = (S_{nu}, S_{na}, D_n, A_n, Q_n, G_n, R_n, A) = (S_{nu}^{**}, S_{na}^{**}, D_n^{**}, A_n^{**}, Q_n^{**}, R_n^{**}, A^{**}) \quad (29)$$

$$\text{And } \lambda^* = \frac{\beta}{N_n^*} (\varepsilon D_n^{**} + A_n^{**}), k_1 = \tau_A + \sigma + \beta_1 + \mu, k_2 = \tau_Q + \delta + \beta_2 + \mu,$$

$$k_3 = m + \mu, k_4 = \alpha + \tau_D + \mu$$

The Drug Endemic Equilibrium (DEE) of the dynamical system in (1-8) can be solved by setting (1-8) to zero. Hence, the endemic equilibrium are;

$$S_{na}^{**} = \frac{\rho S_{nu}^{**} A^{**}}{a\lambda^* + q + \mu} \quad (30)$$

$$S_{nu}^{**} = \frac{\Lambda}{\lambda^* + \rho A^{**} + \mu - \frac{q\rho A^{**}}{a\lambda^* + q + \mu}} \quad (31)$$

$$D_n^{**} = \frac{\lambda^* (S_{nu}^{**} + a S_{na}^{**})}{k_4} \quad (32)$$

$$A_n^{**} = \frac{\alpha}{k_1} D_n^{**} \quad (33)$$

$$Q_n^{**} = \frac{\sigma\alpha}{k_1 k_2} D_n^{**} \quad (34)$$

$$G_n^{**} = \frac{\alpha D_n^{**}}{k_1 k_3} \left(\beta_1 + \frac{\beta_2 \sigma}{k_2} \right) \quad (35)$$

$$R_n^{**} = \frac{1}{\mu} (\tau_D D_n^{**} + \tau_A A_n^{**} + \tau_Q Q_n^{**} + m G_n^{**}) \quad (36)$$

$$A^{**} = \frac{1}{v} \left[\Lambda_1 + \frac{\eta_1 \alpha}{k_1} + \frac{\eta_2 \sigma \alpha}{k_1 k_2} D_n^{**} \right] \quad (37)$$

E. THE BASIC REPRODUCTION NUMBER

The basic reproduction number, also represented as R_0 , is the average number of secondary cases produced when a single infected drug abuser is introduced into a population. It can be used to provide insight into the potential spread of drug abuse. This threshold parameter is crucial for determining whether an infection will establish itself within a population or fade away. The computation of R_0 can be performed using the next-generation matrix approach, initially introduced by [3] and subsequently refined by [21] and [5]. The basic reproduction number serves as a key factor in estimating the stability of the drug-free equilibrium (DFE). If $R_0 < 1$, the drug-free equilibrium is locally asymptotically stable, implying that drug abuse will not persist in the population. Conversely, if $R_0 > 1$, the DFE becomes unstable, allowing for the potential spread of drug abuse

$$R_0 = \rho(FV^{-1})$$

Where ρ denotes the spectral radius (dominant eigenvalue)

Therefore, from the model

$$F_i = \frac{\beta(S_{nu} + aS_{na})}{N_n}(\varepsilon D_n + A_n) \quad (38)$$

$$F = \begin{pmatrix} \frac{\beta\varepsilon(S_{nu} + aS_{na})}{N_n} & \frac{\beta(S_{nu} + aS_{na})}{N_n} \\ 0 & 0 \end{pmatrix} \quad (39)$$

$$\text{Likewise } V_i = \begin{pmatrix} (\alpha + \tau_D + \mu)D_n \\ -\alpha D_n + (\tau_A + \sigma + \beta_1 + \mu)A_n \end{pmatrix} \quad (40)$$

$$V = \begin{pmatrix} \alpha + \tau_D + \mu & 0 \\ -\alpha & \tau_A + \sigma + \beta_1 + \mu \end{pmatrix} \quad (41)$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\alpha + \tau_D + \mu)} & 0 \\ \frac{\alpha}{(\alpha + \tau_D + \mu)(\tau_A + \sigma + \beta_1 + \mu)} & \frac{1}{(\tau_A + \sigma + \beta_1 + \mu)} \end{pmatrix} \quad (42)$$

$$FV^{-1} = \begin{pmatrix} \frac{\beta\varepsilon(S_{nu} + aS_{na})}{N_n} & \frac{\beta(S_{nu} + aS_{na})}{N_n} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(\alpha + \tau_D + \mu)} & 0 \\ \frac{\alpha}{(\alpha + \tau_D + \mu)(\tau_A + \sigma + \beta_1 + \mu)} & \frac{1}{(\tau_A + \sigma + \beta_1 + \mu)} \end{pmatrix} \quad (43)$$

$$R_0 = \rho(FV^{-1}) \quad (44)$$

$$R_0 = \frac{(\nu(q + \mu)\mu(\rho\Lambda_1 + \nu(q + \mu)) + \Lambda_1\mu\rho\alpha(\rho\Lambda_1 + \nu(q + \mu)))\beta(\varepsilon\mu + (\tau_A + \sigma + \beta_1)\varepsilon + \alpha)\Lambda}{\mu(\rho\Lambda_1 + \nu(q + \mu))\mu(\rho\Lambda_1 + \nu(q + \mu))N_n(\alpha + \tau_D + \mu)(\tau_A + \sigma + \beta_1 + \mu)} \quad (45)$$

F. SENSITIVITY ANALYSIS OF MODEL PARAMETERS

In this section, we perform a sensitivity analysis to determine the contribution of each parameter to the basic reproduction number. This analysis determines the contribution of each parameter value to the reproduction number, as reported by [17]. To quantify sensitivity, we employ the normalized forward sensitivity index, also referred to as the elasticity index [18]. This index measures the relative change in R_0 concerning a relative change in a given parameter and is mathematically defined as:

$$S_x^{(R_0)} = \frac{\partial R_0}{\partial x} \times \frac{x}{R_0} \quad (46)$$

Where x represents any parameter in the model.

Table 2: Model Parameters, Baseline Value and Source

Parameter	Value	Source
α	0.3	[10]

ρ	0.7	[16]
Λ_1	10	Estimated
Λ	3	Estimated
ν	0.5	Estimated
q	0.2	Estimated
μ	0.02	[10]
ε	0.02	Estimated
τ_A	0.25	[16]
τ_D	0.01	[16]
σ	0.03	Estimated
β_1	0.3	[2]
β	0.3	[2]

Table 3: Sensitivity Index of the Parameter Values

Parameter	Description	Sensitivity Index
β	Initiation rate	+1.0000
Λ	Human recruitment	+1.0000
α	Progression rate from D_n to A_n	+0.9792
ρ	Rate at which awareness programs move susceptible unaware to susceptible aware	+0.8572
Λ_1	Growth influx of awareness	+0.8572
ν	The time rate of decay of drug addiction prevalence	+0.0696
q	Rate at which S_{na} loses awareness	+0.0633
μ	Natural death	-0.1596
ε	Relative influence of drug users compared to abusers	+0.0385
τ_A	Progression rate from A_n to R_n	-0.4006
τ_D	Progression rate from D_n to R_n	-0.0303
σ	Progression rate from A_n to Q_n	-0.0481
β_1	Progression rate from A_n to G_n	-0.4807

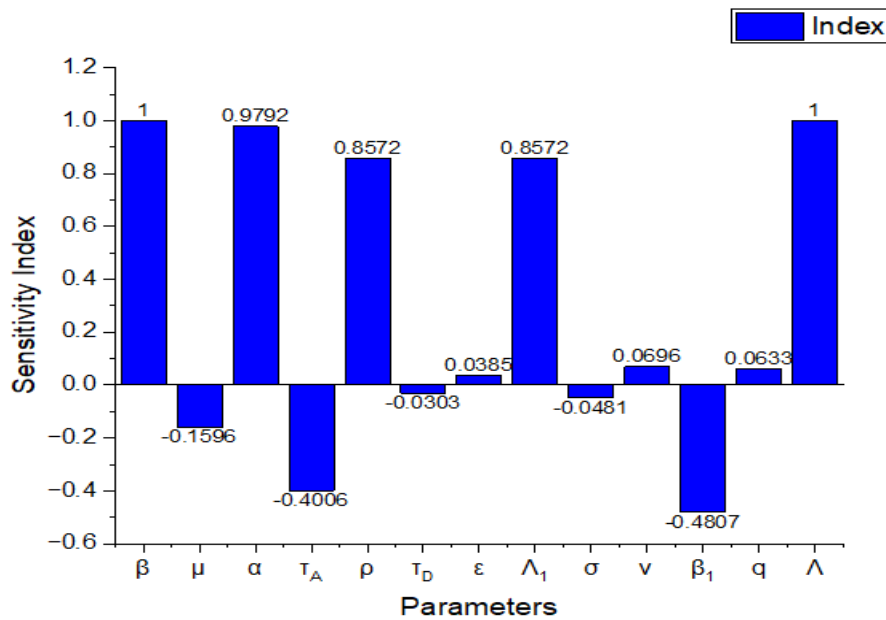


Figure 2: Sensitivity Indices Chart

Table 3 presents the sensitivity indices of the effective reproduction number with respect to the model parameters. The results (sensitivity indices) establish that the drug epidemic is primarily driven by initiation β and human recruitment Λ , both exhibiting the maximum positive index of +1, which underscore primary prevention as the single most essential control strategy. The rapid escalation from casual drug use to chronic abuse α further highlights the necessity for early intervention before addiction fully take hold. On the control side, transitioning active abusers into guidance and counseling programs β_1 serves as the primary protective mechanism against the epidemic, closely supported by movement into recovery τ_A . While awareness parameters (ρ, Λ_1) exert a surprisingly high positive influence within this model structural formulation, secondary factor such as high mortality μ , loss of awareness q , and early recovery from casual use τ_D play comparative minor roles in overall threshold dynamics. Ultimately, public health strategies must balance top-of-funnel prevention to restrict initial entry with robust counselling infrastructure to pull active abusers out of transmission pathways.

G. DISCUSSION OF RESULTS

The section gives a presentation together with an interpretation of the sensitivity analysis of the basic reproduction number, R_0 , with regard to the key model parameters. As the number R_0 determines the stability of the drug-free equilibrium (it shows that drug use cannot persist in the population when R_0 is less than 1), examining how R_0 responds to changes in the parameters helps in identifying the most effective control strategies. As shown in Figure 3, R_0 decreases steadily as the progression rate from drug abusers to disabled addicts (σ) increases, falling from about 0.38 when σ is 0 (with ρ equal to 0.1) to 0.15 when σ is 1. This result is to be expected since the active abusers (A_n) are the main source of the pressure to initiate drug use, while the disabled addicts (Q_n) are removed from the transmission chain; faster progression out of the abuser group reduces the number of people who can transmit the disease and thus leads to a lower R_0 . The figure also

shows that greater awareness (ρ) results in a lower R_0 , the lowest value occurring when ρ is 0.5 and the highest when ρ is 0.1. The fact that the curves for $\rho = 0.5$ and $\rho = 0.3$ are so close together indicates that the returns diminish. Awareness campaigns are most effective when the level of efficacy increases from low to moderate levels, with only limited extra benefit being obtained at higher levels.

As illustrated in Figure 4, R_0 increases almost linearly with the transmission rate of drug initiation (β), going from 0 when β is 0 to about 0.9 when β is 1. This shows that β is the main factor affecting R_0 . The awareness curves are very close to one another, with only small variations at higher values of β (the lowest correlation being $\rho = 0.9$ and the highest $\rho = 0.5$), which proves that when β is high, awareness has only a small effect on R_0 . It follows that it is necessary to reduce β , for instance by limiting the availability of drugs and peer contact, since awareness measures on their own are not sufficient to make up for a high initiation rate.

Figure 5, the basic reproduction number R_0 decreases in a convex manner as the recovery rate from drug users (τ_D) increases, falling from about 0.28 when τ_D is 0 to around 0.10 when τ_D is 1, with the largest decrease occurring at smaller values of τ_D . This shows that gradually moving drug users into the recovered category via treatment, guidance and counselling not only reduces the number of people who use drugs but also lowers R_0 . The order according to ρ is kept ($\rho = 0.5$ being the highest and $\rho = 0.9$ the lowest), although the curves for $\rho = 0.7$ and $\rho = 0.9$ are almost identical once more showing that there is saturation in the benefits of awareness. The results indicate that early intervention with current drug users is very effective, and the initial improvements in treatment and recovery rates produce the greatest reductions in R_0 .

A similar trend is observed in Figure 6 for the recovery rate among drug abusers (τ_A): R_0 decreases in a convex manner from about 0.46 when τ_A is 0 to 0.14 when τ_A is 1. As the abusers (A_n) are the most active transmitters, moving them into recovery has a greater impact on R_0 than the corresponding pathway for drug users (D_n). The curves for $\rho = 0.7$ and $\rho = 0.9$ are almost identical, with $\rho = 0.5$ being slightly higher, once more showing the effect of diminishing returns. The intensive rehabilitation of abusers together with awareness campaigns is one of the most effective control measures.

The trend shown in Figure 7 for the rate at which awareness declines (q) is that R_0 increases steadily and in a concave manner with q , varying from about 0.25 to 0.28 when q is 0 to 0.37 to 0.45 when q is 1, according to the value of ρ . As awareness fades, more people who are susceptible return to the high-risk category and thus the population's vulnerability to drug use increases. The large difference in values between $\rho = 0.1$ and $\rho = 0.5$ when $q = 1$ shows that the advantages of having awareness disappear quickly if it is not continuously maintained. Sustained, repeated campaigns that keep q low are essential, since short-term interventions become ineffective quickly, especially when programme effectiveness is low.

To sum up, these findings show a clear order of control measures. Since the transmission rate β is the main factor affecting R_0 , decreasing drug initiation should be the most important intervention. The rates of recovery and progression (τ_A , τ_D , and σ) all have a effect in reducing R_0 , with the rate associated with the abuser-to-recovered transition (τ_A) having the largest impact. The rate at which awareness wanes, q , is a major risk factor that raises R_0 . Even though awareness (ρ) always leads to a reduction in R_0 , diminishing returns are seen at higher levels, which means that awareness should be used in addition to, not as a substitute for, direct treatment and strategies

aimed at reducing drug initiation. R_0 stays below one for all ranges of the parameters, showing that the drug-free equilibrium is locally stable. Nevertheless, as R_0 approaches one, the decline in drug use slows down and the temporary burden rises. It is necessary to reduce R_0 by increasing τ_A , τ_D , and σ ; decreasing q ; lowering β ; and having a moderately high ρ in order to shorten the epidemic and reduce its peak impact.

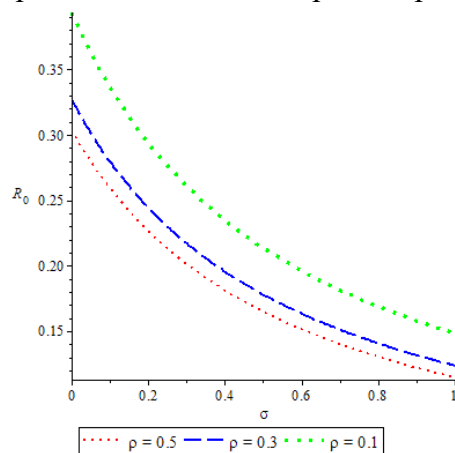


Figure 3: Effect of Awareness on R_0 and Progression Rate from A_n to Q_n

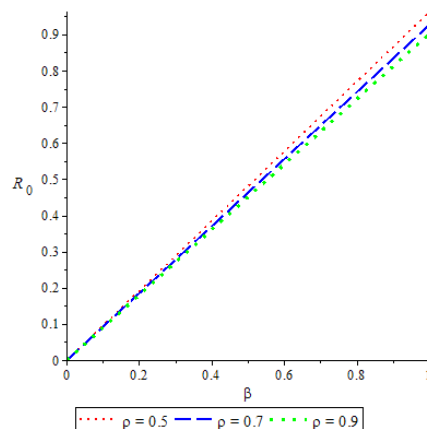


Figure 4: Effect of awareness on R_0 and transmission rate of drug initiation

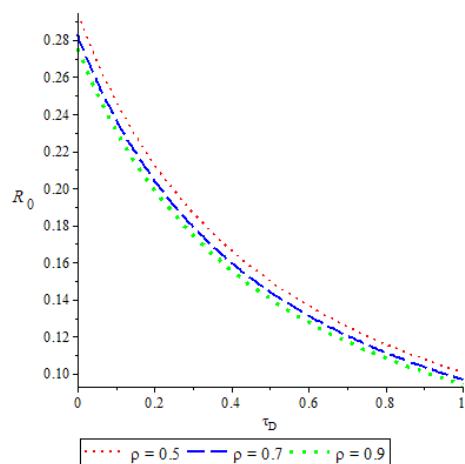


Figure 5: Effect of awareness on R_0 and progression rate from D_n to R_n .

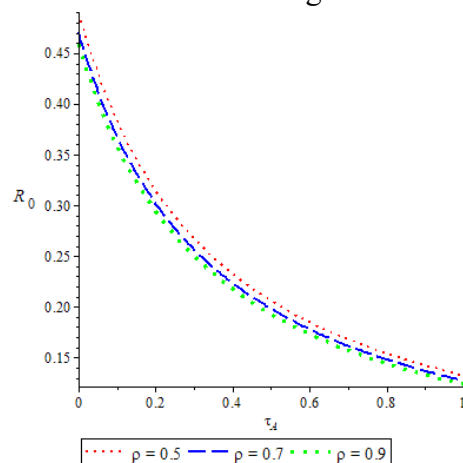


Figure 6: Effect of awareness on R_0 and transmission rate from A_n to R_n

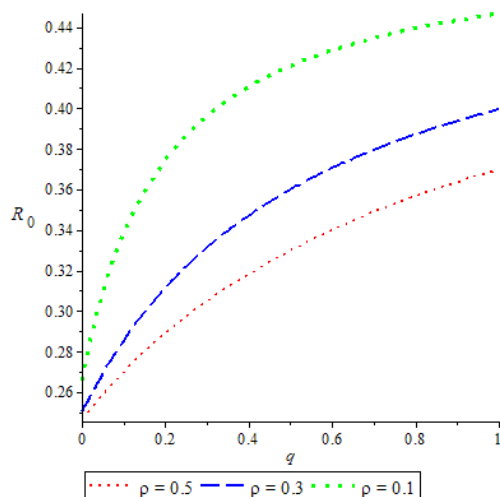


Figure 7: Effect of awareness on R_0 and Rate at which S_{na} lose awareness

III. LIMITATIONS OF THE STUDY

This study has some limitations. The model assumes homogeneous population mixing and deterministic dynamics, which may not accurately represent real-world social interactions. Because epidemiological data are limited, we estimated some parameters, which may reduce the accuracy of the simulation results. As a result of these assumptions and parameter assessments, the qualitative results remain valid, such as the finding that primary prevention reduces initiation and counselling lowers chronic prevalence. However, quantitative predictions, including specific target rates for intervention or precise timelines for eradication, should be regarded as illustrative rather than definitive policy recommendations.

IV. CONCLUSION

In this study, a deterministic mathematical model was developed to evaluate the dynamics of drug abuse in the presence of media-driven awareness programs, guidance and counseling interventions. Comprehensive qualitative analysis confirmed the positivity and epidemiological feasibility of the system, establishing R_0 as the sharp threshold parameter for epidemic persistence versus elimination. Sensitivity analysis revealed that the drug epidemic is overwhelmingly driven by the initiation rate (β) and recruitment (Λ), demonstrating that primary prevention is the single most vital intervention. On the control side, transitioning active abusers into counseling and recovery (τ_A) serves as the primary protective mechanism against population-level spread. Numerical simulations validated the analytical results, showing that while awareness (ρ) consistently reduces R_0 , it exhibits diminishing returns at high coverage levels and wanes rapidly if not maintained. Consequently, effective public health policy must adopt a dual strategy: combining upstream prevention to restrict initial drug entry with robust counseling infrastructure to pull active abusers out of transmission pathways, supported by sustained, long-term public awareness campaigns.

REFERENCES

- [1] Benedetti, F. (2013). Placebo and the new physiology of the doctor-patient relationship. *Physiological Reviews*, 93(3):1207–1246.
- [2] Cecilia, A., & George, B. (2021). A SEIGWR model of drug abuse. *Journal of Mathematical Biology*, 82, 101–120.
- [3] De Jong, M. C., & Diekmann, O. (1992). A method to calculate for computer-simulated infections the threshold value, R_0 , that predicts whether or not the infection will spread. *Preventive Veterinary Medicine*, 12(3–4), 269–285.
- [4] Degenhardt, L., Hall, W., Warner-Smith, M., & Lynskey, M. (2004). Illicit drug use. In M. Ezzati, A. D. Lopez, A. Rodgers, & C. J. L. Murray (Eds.), *Comparative quantification of health risks: Global and regional burden of disease attributable to selected major risk factors* (Vol. 1, pp. 1109–1176).
- [5] Diekmann, O., & van Gils, S. A. (2009). On the cyclic replicator equation and the dynamics of semelparous populations. *SIAM Journal on Applied Dynamical Systems*, 8(3), 1160–1189.
- [6] Edgardo, R. M. (2020). Substance use disorders and their impact on mental health. *Journal of Clinical Psychology*, 76(2), 203–210.
- [7] Edward, J. (2016). *Understanding drug abuse and addiction*. Health Press.
- [8] Eguda, P., et al. (2022). Mathematical model of drug addiction among youths. *Mathematical Biosciences*, 350, 1–12.
- [9] Gogovi, A., et al. (2022). SEIPR model of drug abuse in college students. *Journal of Applied Mathematics*, 79, 230–245.
- [10] Muhammed H., Fatmawati, & Miswanto. (2019). Mathematical model analysis of a drug transmission with criminal law and its optimal control. In *AIP Conference Proceedings* (Vol. 2192, Article 060010). AIP Publishing LLC.
- [11] Massawe, H. (2015). *Drug abuse and its effects in society*. Tanzania Press.
- [12] Mansouri, F (2024). Drug Education and Its Importance in Building a Healthy Society. *J Drug Abuse*. 10:54.
- [13] Momoh, A. A., Alhassan, A., Ibrahim, M., and Amoo, S. (2022a). Curtailing the spread of drug-abuse and violence co-menace: An optimal control approach. *Alexandria Engineering Journal*, 61(6):4399–4422.
- [14] Momoh, S. A., Johnson, A. O., and Bello, M. K. (2022b). The societal consequences of drug abuse in nigerian youths. *International Journal of Social Science Research*, 10(1):45–58.
- [15] Muli, K. (2024). *Mathematical Modelling of Drug Abuse Dynamics*. PhD thesis, University of Nairobi.
- [16] Orwa, T., & Nyabadza, F. (2019). Mathematical modelling and analysis of alcohol–methamphetamine co-abuse in the Western Cape Province of South Africa. *Cogent Mathematics & Statistics*, 6(1), Article 1641175.
- [17] Osman, S., Makinde, O. D., & Theuri, D. M. (2018). Mathematical modelling of transmission dynamics of anthrax in human and animal population. *Mathematical Theory and Modelling*, 8.
- [18] Rodrigues, H. S., Monteiro, M. T. T., & Torres, D. F. M. (2013). Sensitivity analysis in a dengue epidemiological model. In *Conference Papers in Science* (Vol. 2013, Article 721406). Wiley
- [19] United Nations Office on Drugs and Crime (2014), *World Drug Report, 2014, Analysis*. UNODC, United Nations Office on Drugs and Crime (UNODC).
- [20] United Nations Office on Drugs and Crime. (2023). *World drug report 2023*. <https://www.unodc.org/unodc/en/data-and-analysis/world-drug-report-2023.html>. (Accessed: 2025-09-11)

- [21] Van den Driessche, P. & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical biosciences*, 180 (1-2), 29–48.
- [22] Wang, W., Lu, S., Tang, H., Wang, B., Sun, C., Zheng, P., ... & Kang, Y. (2022). A scoping review of drug epidemic models. *International journal of environmental research and public health*, 19(4), 2017.
- [23] World Health Organization. (2011). Global status report on alcohol and health 2011. <https://www.who.int/publications/i/item/9789241564151>.
(Accessed: 2025-09-11)