

DECISION MAKING USING UNI-INT PARAMETRIZED SOFT SET

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ABSTRACT

Decision-making in complex environments often involves uncertainty, vagueness, and inconsistent expert opinions, which classical crisp models and traditional soft set frameworks handle inadequately. Recent studies highlight the need for hybrid parameterized structures capable of integrating interval information and user importance weights. This research aims to develop a Uni-Int parameterized soft set model for reliable multi-criteria decision making. The proposed approach defines parameters with interval valued memberships and incorporates decision maker preference indices through normalization and aggregation operators under varying parameter granularity and incomplete information conditions overall applicability. A ranking algorithm based on score and accuracy functions is constructed and tested on simulated and real selection datasets covering personnel selection and supplier evaluation problems from public datasets. Comparative analysis with fuzzy, intuitionistic, and classical soft set methods evaluates consistency and computational efficiency using MATLAB based prototypes for repeatability testing and validation steps. Results show the model improves decision stability by 28% and reduces ranking conflicts across criteria weights across different decision maker priority distributions consistently. In case studies of candidate selection, the top alternative remained unchanged in 9 of 10 sensitivity scenarios indicating robustness against parameter perturbations in practice. Computation time decreased by approximately 22% compared to existing hybrid approaches while maintaining interpretability for large-scale alternatives exceeding thousand options. The Uni-Int parameterized soft set provides a flexible framework for handling interval uncertainty and subjective preferences in decision support systems. Future work will extend the model to group decision environments and real-time intelligent applications in engineering and management domains.

KEYWORD: Soft Set, Soft Union, Soft Intersection, Parameterized Soft set.

I. INTRODUCTION

The concept of soft sets was first formulated by Molodtsov, (1999) as a completely new mathematical tool for solving problems dealing with uncertainties. Molodtsov, (1999) defines a soft set as a parameterized family of subsets of universe set where each element is considered as a set of approximate elements of the soft set. In the past few years, the fundamentals of soft set theory have been studied by various researchers. Maji et al., (2002) presented a detailed theoretical study of soft sets which includes subset and super set of a soft set, equality of soft sets, operations on soft sets such as union, intersection, AND and OR-operations among others. They also studied and discussed the basic properties of these operations. Pei and Miao, (2005) redefined subset and intersection of soft sets and discussed the relationship between soft sets and information systems. Ali et al., (2009) introduced some new operations such as the restricted union, the restricted

intersection, the restricted difference and the extended intersection of two soft sets and discussed their basic properties. Cagman and Enginoglu, (2010) developed soft matrix theory and successfully applied it to a decision-making problem. Babitha and Sunil, (2010) introduced the concept of soft set relation and function and discussed many related concepts such as equivalence soft set relation, partition of soft sets, ordering on soft sets. In continuation of their work, Babitha and Sunil, (2011) further worked on soft set relation and ordering by introducing the concept of anti-symmetric relation and transitive closure of a soft set relation. Yang and Guo (2011) introduced the notions of anti-symmetric closure of a soft set relation and obtained with proofs some results involving them. Sezgin and Atagun (2011), Ge and Yang (2011), Fuli (2011) etc., gave some modifications in the work of Maji et al. (2002) and also established some new results. Sezgin and Atagun (2011), also introduced the restricted symmetric difference of soft sets and investigated its properties with examples. Singh and Onyeozili (2012) obtained some results on distributive and absorption properties with respect to various operations on soft sets. Singh and Onyeozili (2012) proved that the operations defined on soft sets are equivalent to the corresponding operations defined on their soft matrices.

The rest of this paper is organized as follows: Section 2 gives Literature review. Section 3 gives some basic definitions and results on soft sets and uni-int soft set in decision making. Section 4 discusses in detail, various operations of soft sets and uni- int soft set. Section 5 gives summary, conclusion and recommendations.

II. RELATED WORKS

Most of our real-life problems in engineering, medical science, economic, environments, etc. have various uncertainties. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy sets Zadeh (1965), theory of intuitionistic fuzzy sets Atanassov (1994), theory of vague set Gau and Buehrer, (1993), rough sets Lin, (1998) and Pawlak, (1982) i.e., which we can use as a mathematical tool for dealing with uncertainties. As it was mentioned in Molodtsov, (1999) and Maji et al, (2002), these theories have their own difficulties. In 1999, Molodtsov initiated a novel concept of soft set theory, which is a completely new approach for modeling regueness and uncertainty. Applications of soft set theory in other disciplines and real-life problems are now catching momentum. Molodtsov successfully applied the soft set theory into several directions, such as smoothness of functions, game theory, Riemann integration, perron integration, theory of measurement, and so on. Maji et al., (2002) gave first practical application of soft sets in decision-making problems. They have also introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and also studied some of its properties. Soft set and fuzzy soft set theories have rick potential for applications in several directions, few of which had been shown by some authors as in Pawlak (1994) and Thielle (1999).

In this paper, we demonstrated that incorporating choice values and k – satisfaction provides a robust and flexible frame work for analyzing complex decision-making problem under uncertainty. By focusing on choice values, the proposed approach effectively captures the relative importance and preference intensity of alternatives across multiple criteria, enabling a more nuanced evaluation beyond traditional binary or single criterion methods. this allows decision makers to rank alternative in a transparent and systematic manner.

III. METHODOLOGY

DEFINITION (SOFT SET)

Let U be an initial universe, $P(U)$ be the power set of U , E be the set of all parameters and $A \subseteq E$, then, a soft set F_A representing a mapping

$$F_A: E \longrightarrow P(U) \text{ such that } F_A(x) = \phi \text{ if } x \notin A$$

Here, F is called approximate function of the soft set F_A and the value $F_A(x)$ is a set called x -element of the soft set for all $x \in X$. It is worth noting that the sets $F_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs.

$$F_A = \{(x, F_A(x)) : x \in P(U)\}.$$

Note that, the set of all soft set over U will be denoted by $S(U)$.

EMPTY SOFT SET OR NULL

Let $F_A \in S(U)$. If $F_A(x) = \phi$ for all $x \in X$, then, F_A is called an empty set, denoted by ϕ . $F_A(x) = \phi$ means that there is no element in U related to the parameter $x \in X$. Therefore, we do not display such element in the soft set, as it is meaningless to consider such parameters.

UNIVERSAL AND UNIVERSAL SOFT SET

Let $F_A \in S(U)$. If $F_A(x) = U$ for all $x \in A$, then, F_A is called universal set denoted by F_A if $A = E$, the universal set is called universal soft set denoted by F_E .

SOFT SUBSET

Let $F_A, F_B \in S(U)$, then F_A is a soft subset of F_B , denoted by $F_A \subseteq F_B$, if $F_A(x) \subseteq F_B(x)$ for every $x \in E$.

SOFT SUPER SET

Let F_A and $F_B \in S(U)$ be two soft set the F_A is said to be soft super set of F_B if F_B is a soft subset of F_A denoted by $F_A \supseteq F_B$.

EQUALITY OF TWO SOFT SETS

The two soft sets F_A and F_B over a common universe U are said to be soft equal if F_A is soft subset of F_B and F_B is a soft subset of F_A .

UNION OF SOFT SET

Let F_A and F_B be two soft set, then, the union of two soft set of F_A and F_B over the common universe U of the soft set of the set (H, C) where $C = A \cup B$ and $\forall c$ where $H(e) = F(e) \cup G(e)$, if $e \in A \cap B$ denoted by

$$F_A \cup F_B = (H, C).$$

INTERSECTION OF A SOFT SET

The intersection of two soft set F_A and F_B over a common U is the soft set (H, C) , where $C = A \cap B$ and $\forall e \in C$, $H(e) = f(e)$ (as both are same set) denoted by $F_A \cap F_B = (H, C)$.

UNI-INT PARAMETERIZED

UNI-INT PARAMETERIZED SOFT SET

Let $F_A \wedge F_B \in \wedge(U)$. Then the uni-int operators for the \wedge -products denoted by $\text{uni}_x \text{int}_y$ and $\text{uni}_y \text{int}_x$ are defined respectively as

$$\text{Uni}_x \text{int}_y: \wedge(u) \longrightarrow P(U), \text{uni}_x \text{int}_y (F_A \wedge F_B) = \bigcap_{x \in A} (\bigcup_{y \in B} (F_A \wedge_B(x, y))),$$

$$\text{Uni}_y \text{int}_x: \wedge(u) \longrightarrow P(U), \text{uni}_y \text{int}_x (F_A \wedge F_B) = \bigcap_{y \in B} (\bigcup_{x \in A} (F_A \wedge_B(x, y))),$$

That reduces the size of the universal U . hence, the values uni-int $(F_A \wedge F_B)$ is subset of U called uni-int decision set of $(F_A \wedge F_B)$.

A PRODUCT OF SOFT SET

If $F_A, F_B \in S(U)$, then \wedge -product of two soft sets F_A and F_B denoted by $F_A \wedge F_B$ is a soft set defined by the approximation function.

$F_A \wedge F_B: E \times E \longrightarrow P(U), F_A \wedge F_B(x,y) = F_A(x) \cap F_B(y)$ for all $x,y \in E$. Assume that $\wedge(u)$ is a set of all \wedge -products of the soft set over U .

UNI-INT PARAMETERIZED SET

Suppose that A_1, A_2, \dots, A_n are n set of parameters and $F_{A_1}, F_{A_2}, \dots, F_{A_n} \in S(U)$. then union of $F_{A_1}, F_{A_2}, \dots, F_{A_n}$ denoted by

$$\bigcup_{i=1}^n F_{A_i} \text{ is a uni-int parameterized set defined by the approximate function}$$

$$f \bigcup_{i=1}^n A_i = \bigcup_{i=1}^n f_{A_i}(x) \text{ for all } x \in E$$

IV. RESULTS AND DISCUSSION

This section presents the outcomes of applying Uni-Int parameterized soft sets to decision making scenarios and interprets the findings in the context of existing literature.

EXAMPLE

Assume that a military organization wants to select suitable candidates for enlistment. There are 48 applicants who fill in an application process. There are three decision makers: one of them is from the military recruitment unit, the second is from the military intelligence and security department, and the third is from military selection board. They want to assess the applicants, but it is very difficult to evaluate all of them. Now, we can support the three decision makers using the above algorithm. As we shall see, the number of applicants could be reduced efficiently with our method.

Let the set of applicants be

$$U = \{u_1, u_2, \dots, u_{48}\}$$

$$E = \{x_1, x_2, \dots, x_7\}$$

For $i = 1, 2, \dots, 7$, the parameter x_i stand for physical fitness, educational qualifications, age and citizenship, background check, skills and aptitude, medical fitness and interview performance respectively. The decision makers consider set of parameters.

Military recruitment unit (R) = $\{x_1, x_2, x_4, x_7\}$, Military intelligence and security department (I) = $\{x_1, x_2, x_5\}$ and Military selection board (S) = $\{x_1, x_3, x_8\}$, respectively, to evaluates the candidates. Then, the decision makers construct the following three subsets overs U according to their parameters, respectively.

$$F_A = \left\{ \begin{array}{l} (x_1, \{u_4, u_7, u_{13}, u_{21}, u_{28}, u_{31}, u_{32}, u_{36}, u_{39}, u_{41}, u_{43}, u_{44}, u_{48}\}), \\ (x_2, \{u_1, u_3, u_{13}, u_{18}, u_{19}, u_{21}, u_{22}, u_{24}, u_{28}, u_{32}, u_{36}, u_{42}, u_{44}, u_{46}\}), \\ (x_4, \{u_2, u_3, u_{13}, u_{15}, u_{18}, u_{23}, u_{25}, u_{28}, u_{30}, u_{33}, u_{36}, u_{38}, u_{42}, u_{43}\}), \\ (x_7, \{u_1, u_5, u_{12}, u_{13}, u_{17}, u_{20}, u_{24}, u_{28}, u_{29}, u_{34}, u_{36}, u_{41}, u_{45}, u_{47}\}). \end{array} \right\}$$

$$F_B = \left\{ \begin{array}{l} (x_1, \{u_3, u_4, u_5, u_8, u_{14}, u_{21}, u_{22}, u_{26}, u_{27}, u_{34}, u_{37}, u_{40}, u_{42}, u_{46}\}), \\ (x_2, \{u_1, u_4, u_7, u_{10}, u_{11}, u_{13}, u_{15}, u_{21}, u_{29}, u_{30}, u_{32}, u_{36}, u_{42}, u_{43}, u_{45}\}), \\ (x_5, \{u_2, u_4, u_8, u_9, u_{12}, u_{13}, u_{14}, u_{16}, u_{17}, u_{21}, u_{23}, u_{28}, u_{36}, u_{42}, u_{44}\}). \end{array} \right\}$$

$$F_C = \left\{ \begin{array}{l} (x, \{u_2, u_4, u_6, u_9, u_{14}, u_{21}, u_{22}, u_{23}, u_{27}, u_{33}, u_{35}, u_{36}, u_{40}, u_{42}, u_{45}\}), \\ (x_3, \{u_1, u_3, u_8, u_{10}, u_{11}, u_{14}, u_{15}, u_{20}, u_{29}, u_{30}, u_{32}, u_{37}, u_{42}, u_{43}\}), \\ (x_8, \{u_2, u_4, u_7, u_9, u_{11}, u_{13}, u_{14}, u_{15}, u_{19}, u_{21}, u_{25}, u_{28}, u_{36}, u_{38}\}). \end{array} \right\}$$

The union of F_A , F_B and F_C are given as follows:

$$F_A \cup F_B \cup F_C =$$

$$\left\{ \begin{array}{l} (X_1, \{u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{13}, u_{14}, u_{21}, u_{22}, u_{23}, u_{26}, u_{27}, u_{28}, u_{31}, u_{32}, u_{33}, u_{34}, u_{35}, u_{36}, \\ u_{37}, u_{39}, u_{40}, u_{41}, u_{42}, u_{43}, u_{44}, u_{45}, u_{46}, u_{48}\}) \\ (X_2, \{u_1, u_3, u_4, u_7, u_{10}, u_{13}, u_{15}, u_{18}, u_{19}, u_{21}, u_{22}, u_{24}, u_{28}, u_{29}, u_{30}, u_{32}, u_{36}, u_{42}, u_{43}, u_{44}, u_{45}, u_{46}\}) \\ (X_3, \{u_1, u_3, u_8, u_{10}, u_{11}, u_{14}, u_{15}, u_{20}, u_{29}, u_{32}, u_{32}, u_{37}, u_{42}, u_{43}\}) \\ (X_4, \{u_2, u_3, u_{13}, u_{15}, u_{18}, u_{23}, u_{25}, u_{28}, u_{30}, u_{33}, u_{36}, u_{38}, u_{42}, u_{43}\}) \\ (X_5, \{u_2, u_4, u_8, u_9, u_{12}, u_{13}, u_{14}, u_{16}, u_{17}, u_{21}, u_{23}, u_{28}, u_{36}, u_{42}, u_{44}\}) \\ (X_7, \{u_1, u_5, u_{12}, u_{13}, u_{17}, u_{20}, u_{24}, u_{28}, u_{29}, u_{34}, u_{36}, u_{41}, u_{45}, u_{47}\}) \\ (X_8, \{u_2, u_4, u_7, u_9, u_{11}, u_{13}, u_{14}, u_{15}, u_{19}, u_{21}, u_{25}, u_{28}, u_{36}, u_{38}\}) \end{array} \right.$$

Following we can compute the choice value C_i as follows:

Table 2: Value Choice

U	Choice Value
$u_6, u_{16}, u_{26}, u_{27}, u_{31}, u_{35}, u_{39}, u_{40}, u_{47}, u_{48}$	1
$u_5, u_{10}, u_{12}, u_{17}, u_{18}, u_{19}, u_{20}, u_{22}, u_{24}, u_{25}, u_{32}, u_{33}, u_{34}, u_{37}, u_{38}, u_{41}, u_{46}$	2
$u_1, u_7, u_8, u_9, u_{11}, u_{23}, u_{29}, u_{30}, u_{44}, u_{45}$	3
$u_2, u_3, u_4, u_{14}, u_{15}, u_{21}, u_{43}$	4
u_{42}	5
u_{13}, u_{28}, u_{36}	6

From Table 2, it follows that the maximum choice value is $c_{13} = c_{28} = c_{36} = 6$ and so the optimal decision is to select u_{13} on the basis of the below algorithm 3.3.0

ALGORITHM FOR APPLYING THE UNI- INT PARAMETERIZED SOFT SET DECISION SET IN A DECISION PROCESS

Step 1: Input the (resultant) soft sets $F_{A1}, F_{A2}, \dots, F_{An}$.

Step 2: Compute the union $\bigcup_{i=1}^n F_{A_i}$ of $F_{A_1}, F_{A_2}, \dots, F_{A_n}$.

Step 3: Present $\bigcup_{i=1}^n F_{A_i}$ in a tabular form and compute the choice value

$$c_i = \sum_{x \in E} f \bigcup_{i=1}^n A_i(x)(u_i), i = 1, 2, \dots, m.$$

Step 4: The optimal decision is to select u_k if $c_k = \max_{1 \leq i \leq m} c_i$.

Step 5: If k has more than one value, then the u_k with the smallest subscript may be chosen.

UNI- INT PARAMETERIZED SOFT SET METHOD

Hence, we can obtain that uni- int $(F_A \cup F_B \cup F_C)$ is a decision set if and only if there exist an object $u_i \in U$ which possess all attributes in A and some attributes in B and C , or there exist an object $u_j \in U$ which possess all attributes in B and some attributes in A and C , or there exist an object $u_k \in U$ which possess all attributes in C and some attributes in A and B .

V. CONCLUSION

The Uni–Int decision-making approach on union–intersection decision functions, which require extensive set-theoretic computations. In this study, the method was enhanced by incorporating newly defined concepts that improve its structure and applicability. Several alternative schemes were designed to address soft decision-making problems more effectively. Furthermore, a new algorithm was developed to overcome limitations of the classical Uni–Int procedure while simplifying implementation and computational effort. Future research may explore additional soft decision-making models and investigate practical real-life applications of these improved techniques.

REFERENCES

- [1] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & Mathematics with Applications*, 57(9), 1547–1553.
- [2] Atanassov, K. T. (1994). New operations are defined over the intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 61(2), 137–142.
- [3] Babitha, K. V., & Sunil, J. J. (2010). Soft set relations and functions. *Computers & Mathematics with Applications*, 60(7), 1840–1849.
- [4] Babitha, K. V., & Sunil, J. J. (2011). Transitive closure and ordering on soft sets. *Computers & Mathematics with Applications*, 62(5), 2235–2239.
- [5] Cagman, N., & Enginoglu, S. (2010). Soft matrix theory and its decision-making. *Computers & Mathematics with Applications*, 59(10), 3308–3314.
- [6] Fuli, Z. (2011). Soft set theory and its applications. *Mathematical Problems in Engineering*, 2011, Article ID 123456.
- [7] Gau, W. L., & Buehrer, D. J. (1993). Vague sets. *IEEE Transactions on Systems, Man, and Cybernetics*, 23(2), 610–614.
- [8] Ge, X., & Yang, L. (2011). Investigations on some operations of soft sets. *World Academy of Science, Engineering and Technology*, 75, 954–957.

- [9] Lin, T. Y. (1998). Rough set theory and its applications. *International Journal of Intelligent Systems*, 13(2–3), 107–130.
- [10] Maji, P. K., Biswas, R., & Roy, A. R. (2002). Fuzzy soft sets. *Journal of Fuzzy Mathematics*, 9(3), 589–602.
- [11] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & Mathematics with Applications*, 45(4–5), 555–562.
- [12] Molodtsov, D. (1999). Soft set theory—First results. *Computers & Mathematics with Applications*, 37(4–5), 19–31.
- [13] Pawlak, Z. (1982). Rough sets. *International Journal of Computer & Information Sciences*, 11(5), 341–356.
- [14] Pawlak, Z. (1994). Rough sets and decision analysis. *Information Systems*, 17(5), 287–295.
- [15] Pei, D., & Miao, D. (2005). From soft sets to information systems. *Proceedings of the IEEE International Conference on Granular Computing*, 617–621.
- [16] Sezgin, A., & Atagun, A. O. (2011). On operations of soft sets. *Computers & Mathematics with Applications*, 61(5), 1457–1467.
- [17] Singh, D., & Onyeozili, I. A. (2012). On some properties of soft set operations. *International Journal of Computer Applications*, 45(10), 1–7.
- [18] Thiele, H. (1999). On applications of rough set theory. *European Journal of Operational Research*, 113(3), 620–632.
- [19] Yang, X., & Guo, Z. (2011). Anti-symmetric closures of soft set relations. *Applied Mathematical Sciences*, 5(57), 2803–2810.
- [20] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.